Tail Risk Hedging: Using Black Swan Events to Generate Returns

BSAMC, Research, Strategic Evaluation

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Abstract

Following the COVID-19 global lockdowns in early March, the abrupt market selloff challenged many asset managers, including renowned investors. With stocks, commodities (oil, silver, and gold), and bonds (apart from U.S. treasuries) falling simultaneously, asset managers had to question the validity of established asset correlations and the status of assets seen as a haven. Moreover, as investment strategies aimed at performing well in any environment performed poorly, many investors have started to look at tail-risk hedging as a better alternative for protecting portfolio downsize during extreme market events.

In this paper, we will first take a closer look at the meaning of tail-risk and how investors can use derivatives such as options, futures, or other instruments to engage in tail-risk hedging in order to improve portfolio performance and generate absolute returns.

Then, we will analyze the historical performance of a 100% allocation to tail-risk and compare it with partial allocations. Also, we will compare the performance of tail-risk investing in strategies such as Trend, 60/40, and risk-parity during different scenarios. In addition, we will also consider the main critics of tail-risk hedging, which are mainly associated with timing, high overall insurance costs during times of uncertainty, and lower long-term performance.

In the third part of the paper, we will dive into the most popular tail-risk strategies using options and look to assess their investment performance and hedging effectiveness. Lastly, we will analyze which tail-risk allocation seems to be the most effective for the long-term performance of a portfolio.

1. Introduction to Tail Risk hedging

1.1. What is Tail Risk?

'Tails' refer to the end portions of the probability distribution curves, which should be bell-shaped if the probability distribution is a Normal distribution. The probability distribution shows a range of potential outcomes that could occur and links directly to that outcome a probability (P). Applying the probability distribution in the case of investing, bell curves represent the likelihood graphically for an investor of achieving a specific return over a predetermined time period. More specifically, under a Normal probability distribution, the most likely returns tend to be highly concentrated around the center, which in that case shows the average expected return (mean). On the other hand, the further the outcome is from the mean, the lower the probability of their occurring. Since the far left and the far-right tails represent the lowest return (Losses) and the highest returns (gains), respectively, asset managers' priority in diversifying their portfolio is to minimize left tail risk without curtailing right tail growth potential.

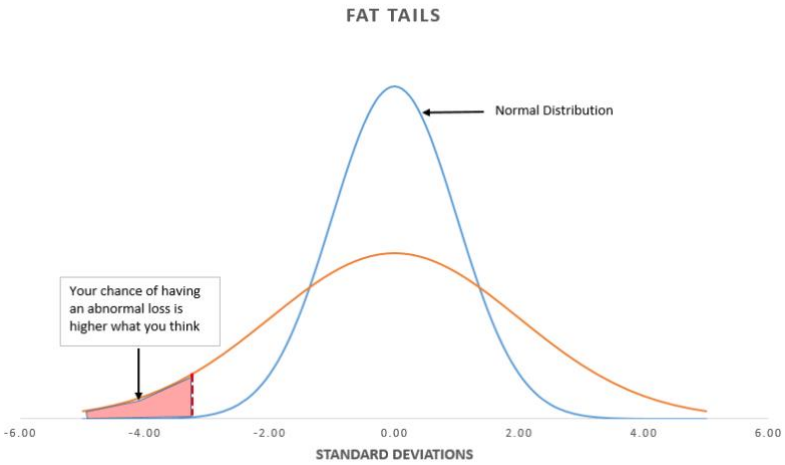


***Figure 1**. Normal Distribution vs. "Fat Tail" Distribution*

Traditionally, asset managers tend to structure their portfolio strategies relying on the main assumption that expected returns can be modelled using the Normal distribution. However, markets don't behave *normally* as the probability of stock swings with a $\sigma > 3$ is higher than the one expressed by the normal distribution, hence a distribution with *fatter tails* (represented by the orange line in Figure 1) is more relevant when modelling actual stock market performance. To further reinforce the relatively frequent occurrence of very rare unpredictable negative events - aka *Black Swan* events - we should consider historical market crashes such as the 1987's Black Monday, the 2000's Tech Bubble burst, the Lehman Brothers' collapse which led to the 2008 financial crisis, the 2010 Flash crash or the 2020's COVID 19 pandemic. In the same way, many studies have concluded, that over the past three decades, market selloffs have occurred approximately every 3 to 5 years, with more dramatic shocks every 10 years. As a consequence, there is clear empirical evidence that the probability distribution of the expected stock returns tends to have *fat tails*.

Given the limitations of the Gaussian distribution, financial literature has tried to look for alternative distributions. For instance, it's graphically shown that T-students distribution associates higher probabilities to extreme events compared to those computed by a Gaussian distribution. As shown in Figure 2, the Normal Distribution underestimates the event probability both close to the mean and close to the tail. In contrast, T-student's distribution can be considered a more reliable way to approximate empirical data. In particular, looking at Figure 3, we see that the standardized t-distribution (the orange dotted line) includes more observations close to the mean and on the tails

compared with the normal distribution. Consequently, T-distribution corresponds more closely with the actual stock performance (shown in the graph on the left). Consequently, using the T-distribution is more reliable for identifying tail risk and consequently engaging in responsible hedging.

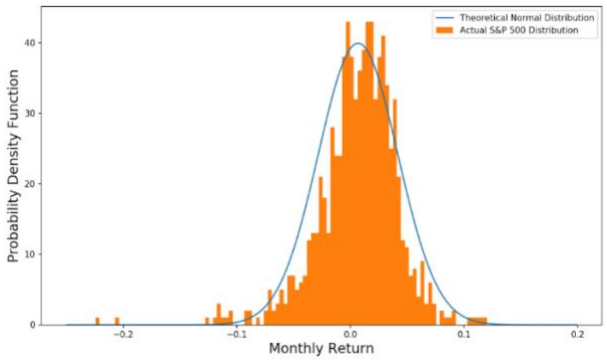


***Figure 2**. Normal Distribution vs. Actual Returns*

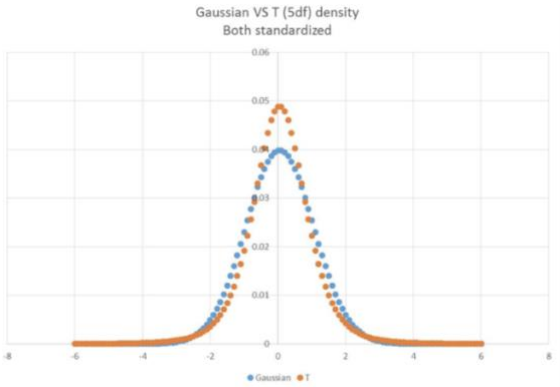


***Figure 3**. Normal Distribution vs. T-Distribution*

1.2. Options and Option-Based Tail Risk Hedging Performance for Portolios

Options are a type of derivative product. Essentially, they are contracts between two parties that give holders the rights, and not the obligation, to buy or sell the underlying asset (equities, bonds, interest rate, currency, ETFs) at a certain price (strike price) within a specific amount of time. The value of the option is tied to the underlying asset, and that is the main reason why options are derivatives: their value derives from the performance of the underlying asset, whatever it is.

There are two types of options: **calls** and **puts**. A **call** option gives the owner the right to buy the underlying asset, while a **put** option gives the owner the right to sell the underlying asset. An option trade always has two sides: a buyer and a seller. As a consequence, the seller of the option (known as the writer) is obliged to buy (in case of a **put** option) or sell (in case of a **call** option) the underlying security if the buy of that contract decides to exercise the option.

Depending on the strike price, a **call** option is defined as:

1. '*In-the-money **call** option*' → strike price is lower than the market price
2. '*Out-of-the-money **call** option'* → strike price is higher than the market price
3. *'At-the-money **call** option'* → strike price equals the market price

The exact opposite is true for **put** options.

Options are essential for hedging tail-risk, and in section 3 of this paper, we explain how investors can implement different strategies using options. For now, it is important to point out the most popular option based tail-risk strategy, which resumes buying OTM **puts**. According to an analysis of AQR carried out in the middle of the Covid-19 crisis, investors tend to acquire and buy OTM **puts** in order to hedge their portfolios during an impending crisis. While over the long-run buying **puts** to hedge is more costly for investors compared to other hedging strategies such as multi-asset class trend-following, buying puts is highly efficient in hedging tail events. In addition, investors can employ more complex option strategies such as stock option collars to minimize the cost of hedging.

Finally, the question that inevitably arises is whether hedging tail-risk adds value to the portfolio performance, compared to other forms of hedging. By comparing the performance of the just mentioned portfolio with the classic 60/40 investment strategy, we can observe that implementing a tail risk hedging strategy allows the investor to **outperform** the 60/40 over a 3-year period.

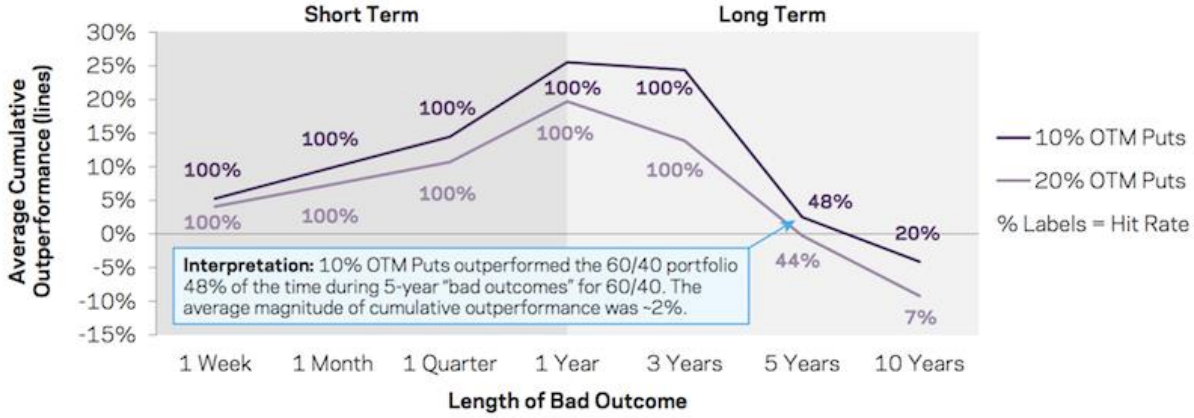


Figure 4. The outperformance of the options during Bad Outcomes for US 60/40 portfolio, January 5, 1996 – March 31, 2020.

But while some investors try to hedge against tail risk, many institutional investors aim to reduce as much as possible the risk of their portfolio while maintaining the long term returns as high as possible. Such strategies include investing in *defensive stocks*, *risk-parity,* and *Trend following (Trend).* Regardless of the strategy, we have to consider the time frame of the performance, and as we can see from figure 5 **10%, OTM puts** have clearly outperformed the classic 60/40 and other strategies too for periods shorter than 1 year. However, the performance advantage eroded for bad stock market periods longer than 1 year, reinforcing the fact that tail-risk performs well if implemented before abrupt, quick market shocks.

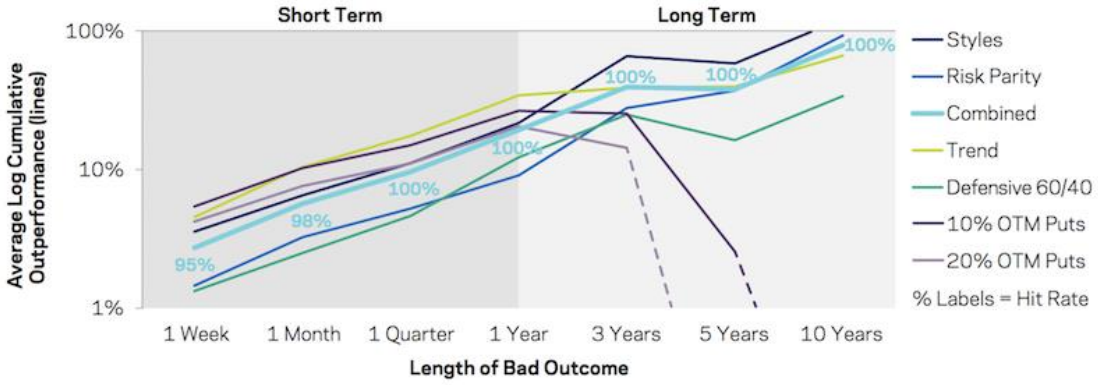


Figure 5*. Outperformance during Bad Outcomes for US 60/40*

2. Comparison Between Tail Risk Hedging and Other Strategies

2.1. 100% Tail Risk Allocation and Partial Risk Allocation

While most trading models assume a normal distribution of events, if this hypothesis were to be true, we would only have massive financial market shocks once in 1000 years. However, about every ten years, an extreme event occurs and rattles the financial markets, showing that in reality, the Gaussian distribution used has fat tails, which account for the fact that tail events occur more often.

For this reason, in this section, it seems reasonable to explain how investors might hedge their positions from extreme events through 100% tail risk allocation or, alternatively, partial allocation strategies. The common view that 100% tail risk allocation strategies cost more but are more effective tail hedges contains a kernel of truth but does not capture the full story.

The 1987 experience taught market participants that in the gapping market falls, option- based protection strategies are more reliable than dynamic strategies, such as portfolio insurance, which depend on the ability to trade continuously.

On the other hand, there is the question of whether the tail insurance provider will be around when you want to collect after a crash event, making counterparty risk an important consideration when talking about financial catastrophe insurance. In addition, there is the question of whether you will still be around (paying those costly tail insurance fees) when the next crash event materializes, the so-called "investor impatience."

In fact, tail hedging strategies should ideally not be viewed as standalone, but rather in conjunction with the portfolio they are supposed to hedge (or insure, or protect, against the worst tail outcomes). But most real-world investors, even those who believe in portfolio perspective and patience:

1. cannot fully resist line-item thinking
2. mainly judge performance after they invested into a fund (at most giving partial credit for earlier wins such as 2008, even if they are part of a public audited track record
3. will find it difficult to stick with a strategy that underperforms more than five years

In reality, the standalone performance of option based tail hedges may have involved a decade or more (2009-19) of not just underperforming but of spending most, if not all, of the capital allocated to them. Whether it is fair or not, the high bleeding costs of 100% tail risk allocation strategies make it less likely that investors will even achieve the long-run returns or tail rewards depicted above; capitulating before the protection event occurs is an all-too-plausible outcome. This return and patience advantage is why people should favor partial risk tail allocation over direct (option-based) tail hedges, more in general, 100% tail risk allocation strategies.

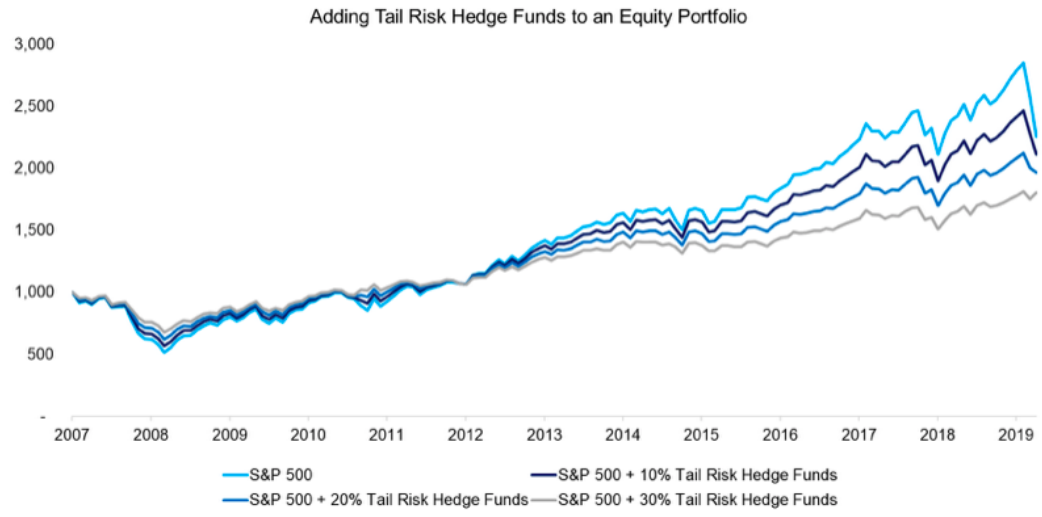
An optimal solution suggested by Universa, an investment management firm that has specialized in risk mitigation since it was founded in 2007, is to allocate 3.33% to its tail-fund, then a partial tail risk allocation is adopted, which we have already said to be preferred with respect to a 100% tail risk allocation, and 96.67% to S&P 500 Index fund. This amount has to be rebalanced, perhaps annually, due to the nature of the strategy, which loses most of the time. Thus, effectively, investors end up paying the "insurance premium" most of the years and hope to mitigate the risk of fat-tails. Its CIO, Mark Spitznagel, argues that this is the long-term wealth maximizing solution and presents a table comparing different approaches. Through this table is pointed out how in the major moments of crisis, such as the

financial crisis of 2008 and March 2020, the most negative period for the world stock exchanges due to the "true beginning of Covid-19", can perform with more than excellent performance compared to the terrible negative performance resulting from other strategies. Therefore, despite the fact that Universa's strategies are performing more on average than the alternatives presented, outstanding performance is implemented in high volatility periods; this implies that Universa's risk mitigation strategies focus on maximizing convexity, are perfectly realized.

		Risk mitigation scorecard			
STRATEGY	**Jan-March 2020**	**2019→March 2020**	**2015→March 2020**	**2008→March 2020**	**2008 March-December**
Universa Tail Hedge (3.33%) + SPX (96.67%)	**0.40%**	**16.20%**	**8.30%**	**11.50%**	**9.90%**
CBOE Eurekahdge Tail Risk (3.33%) + SPX (96.675)	-11.40%	5%	6.30%	7.70%	-29.40%
Ishares 20Y + Treasury (25%) +SPX (75%)	-6.80%	12.20%	7.30%	8.90%	-15.10%
Ishares 3-7Y Treasury (25%) + SPX (75%)	-8.30%	6.90%	6.10%	7.20%	-21.10%
CBOE Eurekahedge Long Volatility (25%) + SPX (75%)	-2.30%	10.70%	6.10%	8.20%	-13.80%
Gold (25%) + SPX (75%)	-9.10%	8.40%	6.50%	7.40%	-25.40%
Hedge Found Index (25%) + SPX (75%)	-10.30%	4.40%	5.60%	6.70%	-27.50%
CTA index (25%) + SPX (75%)	-8.70%	5.60%	5.30%	6.60%	-21.50%
SPX (100%)	-29.00%	9.10%	15.50%	65.70%	-30.70%

Table 1. *Performance of different tail risk allocations*

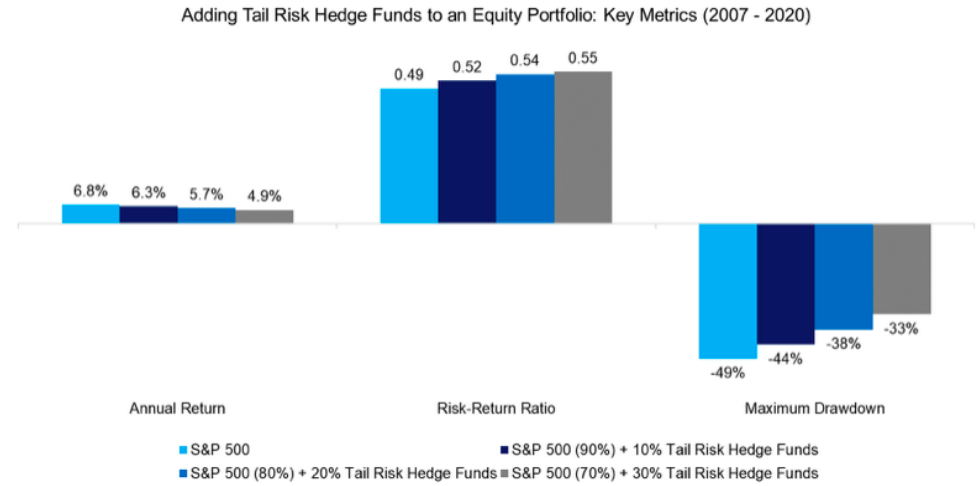
Moreover, the graph below shows that the larger the allocation to tail risk funds, the lower the total return in the period from 2007 to 2019. However, we can also observe that the drawdowns during the global financial crisis in 2008 were reduced, which fulfils the core objective of tail risk funds.



Source: Eurekahedge, FactorResearch

Figure 6. *Equity and Tail Risk hedge*

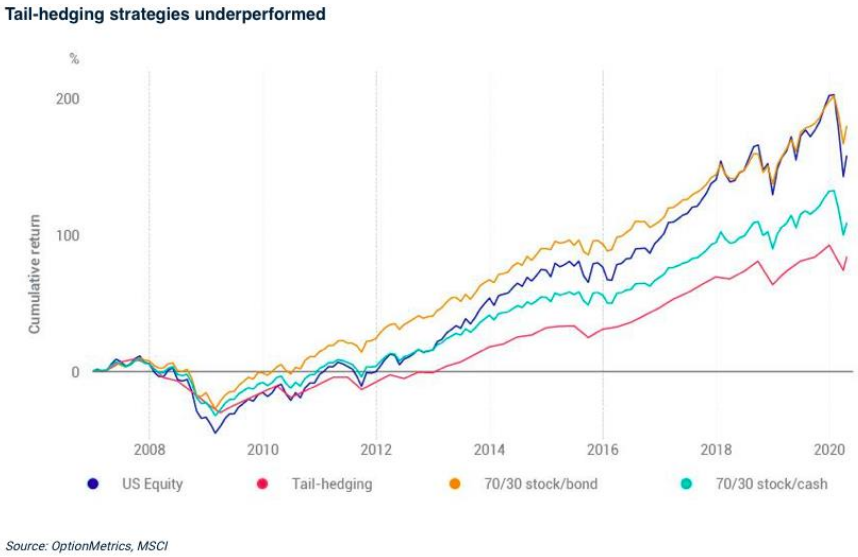
Suppose we switch the perspective to risk-adjusted returns, which highlights that the larger the allocation to tail risk funds in an equity portfolio, the higher the risk. The pain-to-gain ratio, which is defined as the return divided by the maximum drawdown, underlines a moderate increase in risk-adjusted returns. Again, the purpose of tail risk hedging strategies is sufficiently realized.



Source: Eurekahedge, FactorResearch

Figure 7. *Equity and Tail Risk hedge key performance metrics*

At this point, we have widely discussed tail risk hedging performance without pointing out that these kinds of strategies are considerably costly unless they are implemented right before a crisis. The chart in Figure 8 compares the simulated performance of U.S. equity alongside alternatives to reduce tail risk: a zero-cost collar tail-hedging strategy and simple reductions of risk exposure with a 70%/30% stock/cash or bond allocation. The tail-hedging strategy directly insured against tail risk, while the 70/30 strategies reduced tail risk through a reduction of risk exposure more generally, and the 30% bond strategy further benefited from a flight to quality in the Treasury when equity declined. The alternative strategies all avoided the worst of equity's losses, but the tail hedging came at a much steeper cost.



Source: OptionMetrics, MSCI

Figure 8. *Performance of different risk mitigation strategies*

The performance of other tail-hedging strategies was similar to the one shown in the previous graph: the protection provided was more than offset by their cost. Simple strategies to reduce risk overall had similar or smaller drawdowns, while the high costs of maintaining tail insurance eroded performance.

Various tail-hedging strategies underperformed simpler risk-reducing portfolios

Strategy	Average Return	Maximum Drawdown	YTD Return	Sharpe Ratio
MSCI USA Index	8.3%	50.6%	-14.7%	0.48
70/30 equity/bond	8.2%	33.9%	-6.2%	0.72
70/30 equity/cash	6.1%	38.0%	-10.2%	0.48
Equity + Collar (10% downside)	5.1%	36.0%	-4.6%	0.40
Equity + Collar (15% downside)	5.9%	40.6%	-9.5%	0.37
Equity + Collar (20% downside)	6.9%	44.1%	-14.8%	0.39
Equity + Puts (10% downside)	5.4%	43.4%	-9.3%	0.34
Equity + Puts (15% downside)	5.8%	45.1%	-13.0%	0.33
Equity + Puts (20% downside)	6.3%	46.7%	-17.2%	0.34

Source: OptionMetrics, MSCI

Table 2. *Performance metrics for strategies shown in Figure 8*

Historically, tail-hedging strategies are designed to add value during a crisis, but at what cost?

2.2. A More Objective Outlook

2.2.1. Recent Performance and Criticism

In recent months, because of the steep downturn taken by global economies due to the COVID-19 crisis, the tail risk industry has gained a lot of attention for its spectacular profits. Firms like LongTail Alpha, Saba Capital Management, Capstone Investment Advisors, and Nicholas Taleb advised Universa investment have seen their returns skyrocket.

In order to better understand the performance of these funds, it is useful to refer to the CBOE Eurekahedge Tail Risk Hedge Fund Index. This index is the most reliable source regarding Tail Risk Hedging returns as it is composed of the eight most important Tail Risk Funds such as South Capital Advisors and Capstone Investment Advisors.

In 2020 the CBOE Eurekahedge index has gained 14% in February alone and, combined with March performance, is estimated to be up between 32% and 41% for the year. These returns must then be compared with the average stock focused hedge fund performance that, as estimated by the Barclay Hedge Fund Index, is up only about 1,98% for the year, with an average 15.37% yearly gain for the S&P 500 as of December 1, 2020. But to be more explicit, in Table 3, we will show the performance of the main tail risk funds.

Tail Risk Hedge Fund	Performance
LongTail Alpha	Return on the invested premium of 929% as of April 2020
Saba Capital Management Lp	The tail risk fund gained more than 175% through March 2020
Cambia Tail Risk	Profits are up about 25% for 2020
Universal investments	Reported gains of 3600% in February 2020

Table 3. *Tail Risk hedge funds performance during COVID-19 lockdown months*

As highlighted by the former examples, hedge funds that focus on tail risk have booked massive profits during the COVID 19 pandemic. These incredible returns show how well this kind of insurance-like funds can perform during times of stress in financial markets. As volatility soars and liquidity disappears, the options these funds hold a dramatically increase in value, providing a hedge against the declining value of stocks.

Although these numbers are certainly impressive, to better understand the real performance of Tail Risk Hedge Funds, these returns should be analyzed in a longer-term perspective. Indeed, by examining the performance of the CBOE Eurekahedge index from inception, we can clearly see how the returns generated by the index were almost constantly negative and accumulated an overall 32,65% loss as of October 2020. The reason for these negative returns is the fact that this type of investing strategy only generates positive returns in very short time periods and tends to generate negative returns for the remaining time.

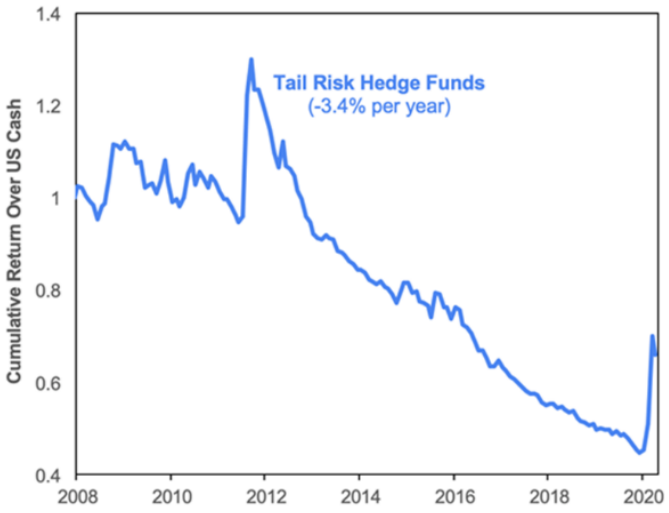


Figure 9. *CBOE Eurekahedge Index*

This chart shows that buyers of tail risk funds have historically paid about 3.4% per year in order to male 25%-50% during the crisis.

In conclusion, it is of paramount importance to understand that, even if this investment strategy can bring phenomenal profits in a short amount of time, in the long term, it tends to be a losing approach if not done correctly. In order to achieve optimal profits, the tail risk approach should be introduced in a diversified portfolio, and the asset allocation should be close to minimal.

As demonstrated in the former paragraph, one of the main drawbacks of tail risk investment is that in the long term, costs tend to outweigh benefits. This is due to the fact that this investment strategy is aimed to be an insurance against adversity, and in order to work, it has to generate high enough returns during stressful times to outweigh the accumulated losses during times of economic growth. Economic calamities such as the COVID-19 or the subprime mortgage crisis are far from common; the profits accumulated during those times, even if enormous, tend to be dwarfed in comparison to the losses accumulated during times of economic growth.

Another major drawback of this investment strategy is the fact that during times of crisis, the tail risk hedging costs tend to skyrocket. This is because as soon as investors realize the incoming risks, they will increase the demand for

derivatives such as put options, which will push the cost of hedging higher and eventually make late hedging uneconomical. Consequently, timing is key for tail risk hedging, and, given that this is hard to get right, investors should look for a cost-effective long-term tail risk hedging strategy.

2.2.2. Tail Risk Hedging and Risk Parity

The risk parity strategy is proposed as an alternative to traditional asset allocation: portfolio shares, allocated to different asset classes, are adjusted on the basis of volatility. In some cases, leverage is also used to balance the various contributions to the overall volatility of the portfolio. The composition of a Risk Parity strategy needs to be periodically rebalanced in response to changes in the expected volatility of the underlying asset classes. These changes shift in the composition of the portfolio, which equalizes the asset class risk contributions, as well as the quantity of leverage that has to be applied to the portfolio in order to target a fixed level of volatility. Specifically, when the volatility of the unleveraged portfolio declines, the quantity of leverage applied increases (and vice versa when the volatilities increase). However, leverage can be very costly in historical periods characterized by relatively high-interest rates. Be aware, risk parity focuses on performing fairly well in all different kinds of economic environments, and this is the main difference with respect to tail risk hedging strategies, which focus on expected tail loss and ought to outperform only in periods of a severe crash.

Let's have a look at a theoretical sample: a 60/40 stocks/bonds portfolio is not well diversified because stocks are more volatile than bonds and will disproportionately dominate the price movement of a portfolio. With this traditional portfolio allocation, equities comprise 90% of the portfolio risk. Historically, equities have had three times the volatility of fixed income securities. The higher equity volatility overtakes the diversification benefits of the bonds. Moreover, bonds yield less than stocks. In much of the developed world, they yield nothing and sometimes negative in both nominal and real terms. Hence, having 40 percent bonds as part of the allocation drags down long-run returns. A risk parity implementation will often get around this problem by leveraging the bond portion of a portfolio to match the risk of the stock portfolio. This will usually be done using bond futures, where a small collateral outlay provides exposure to a larger quantity of bonds. This way, your search for diversification is no longer constrained by its impact on your long-run returns.

Risk parity can make money in periods of drawdowns inequities if other components of the portfolio rise enough to offset the drop. It shouldn't necessarily be expected, as equities are part of the portfolio. But during bad periods for traditional equity-based investors, risk parity has tended to outperform due to its better diversification (and smaller equity allocation). The outperformance is shown up in the strategy below, which covers only liquid asset classes for simplicity.

Its criticism is that during the recent downturn caused by the pandemic, stocks and bonds prices became positively correlated. In this case, risk parity failed to prevent losses streaming from both the stocks bets and the leveraged bonds positions. Figure 11 serves as a reminder that while the negative correlation between the 20+ Treasury ETF and the S&P 500 is a historical norm, recently or more exactly after the spring of 2015, this inverse relationship had broken down to some degree.

Figure 10. *ETF end-of-day closing prices: Bonds and the S&P 500*

To further reiterate the weakness of risk parity in hedging tail risk, we could take a look at Table 4. Here we can clearly see that Risk Parity had positive returns only in 4 out of the 8 tail events. While in the other 4 situations, risk parity lost money, it had a better performance than the SPY, and hence we can conclude that while risk parity is not a reliable tail risk hedge, it can be an effective way to decrease portfolio downsize during bad times and consequently boost long term returns.

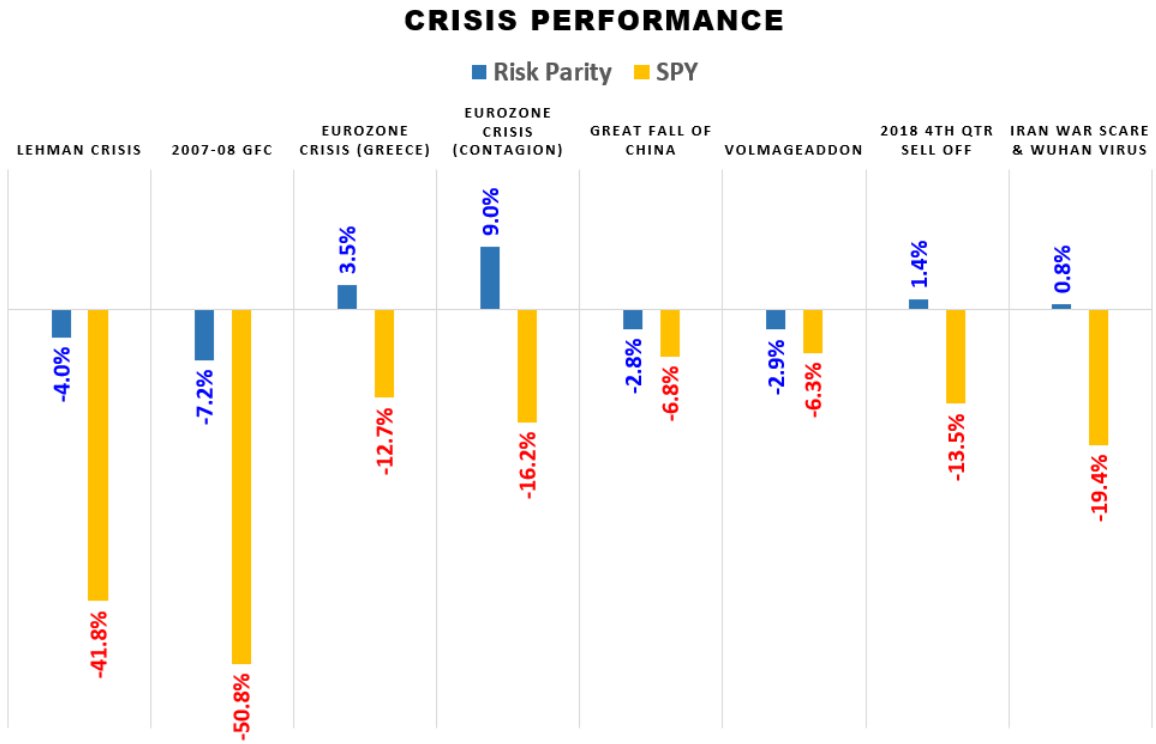


Table 4. *Risk Parity vs. S&P 500 during the crisis*

3. Present and Future Sustainability Practices

3.1. Hedging Tail Risk – Naked Long Puts

Strategy Breakdown:

Let us assume that an investor whose portfolio is fully invested in a particular stock market index named ABC is willing to hedge his portfolio against tail risk. The classic way to proceed would be to buy a naked put option to hedge against a market selloff.

If the investor owns 100 ABC index units, which trade right now at $10 per unit, he can buy for 2 ATM puts for $1 each (0.5 delta*) and a strike price of $10 to fully hedge his portfolio. As each option contract represents 100 shares, the investor's hedge cost will be 2*$1*100=$200. Therefore, the investor's upside potential is the difference between the gains delivered by his long position in the ABC index minus the cost of the put option ($200), and therefore it is unlimited given that ABC's value can grow to infinity. On the other side, the investor's maximum loss is represented by the cost of the put option, as any drop in the price of index ABC below the strike price will be compensated by an equal increase in the value of the option.

**in option pricing, delta represents the increase in the price of a put option for $1 increase in the price of the underlining; therefore, for a full hedge, delta has to be 1 = 0.5 + 0.5*

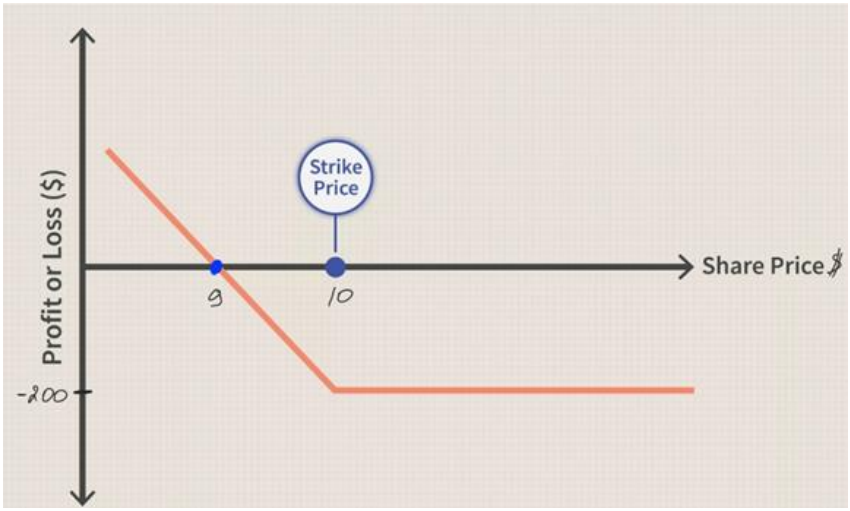


Figure 11. *Long Put Position*

Usually, this basic strategy is implemented using OTM puts, which trade at lower premiums but have a lower delta, so to fully cover a portfolio, an investor will need to buy more contracts. This way, the investor will only protect his portfolio against extreme market swings – which have a standard deviation higher than 3 – and therefore engage in tail risk hedging.

Strategy Benefits:

Depending on the strike prices and the number of contracts bought, this strategy can provide a full hedge against a fall in the value of the ABC index. Also, this strategy can be adjusted to the only hedge partially or to only protect against large negative swings.

Strategy Limitations:

Firstly, tail-risk events, also known as "black swans," tend to be very hard to predict, and therefore investors will need to buy put options with very long expiration cycles that have higher premiums and are trading in less liquid markets. Secondly, buying more liquid options such as three months puts will decrease the overall performance due to the theta decay - which represents the option's value decay as it approaches the expiration day, now sooner than in the case of long-term puts. Also, in the case of slow market downturns, the value of the premium paid for the puts will increase the more the market becomes aware of the existing risks, making it very costly to renew the puts every three months.

3.2. Hedging Tail Risk – Zero Cost Dollar

Strategy Breakdown:

The zero-cost collar is an option-based strategy that can be used as a tail risk hedge. This strategy is attractive for an investor who has owned for a while a stock that has had massive gains but right now, he fears that the stock might go down. Instead of selling right away the stock (in order to still be able to receive some dividends), the investor can choose to implement a zero-cost collar by selling a covered call at a strike price higher than the current stock price and buying a put at a strike price lower than the market.

For example, if the investor owns 100 shares of the underlying stock ABC, which right now trades at $120, the investor can buy a put option for $0.9 at a strike price of $110 and sell a call for $0.9 at a strike price of $128. This way, the investor will pay 100*$0.9 for the put and receive 100*$0.9 for selling the call, leading to a zero-cost hedge.

In this strategy, the investor has capped both his maximum gain and maximum loss. When the price goes above $128, the investors are assigned (must fulfill the obligation he has as a call seller) and hence must sell his shares, capping his extra profits at ($128-$120)*100= $800. In contrast, if the price drops below $110, the investor will have a maximum loss of ($120-$110)*100=$1000 as he has the right to sell his shares at 110 no matter how low the market price.

As a remark, if the investor is willing to increase his maximum profit, he can apply the same strategy, this time selling OTM calls while buying ATM puts. Of course, as the OTM calls will pay him less than the amount he needs to pay for the ATM puts, he will incur a credit, and so the collar will not be zero-cost anymore.

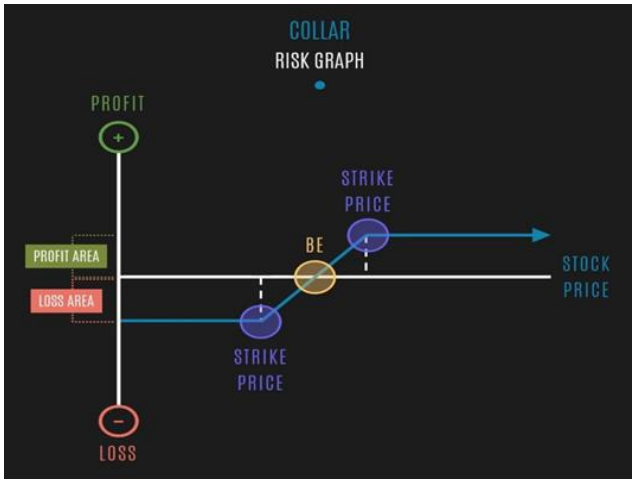


Figure 12. *Zero Cost Collar*

Strategy benefits:
The zero-cost put collar can hedge extreme market sell-offs as it caps both maximum gains and losses at a certain level with low or close to zero costs. Therefore, if used at the right timing, it can be very useful to hedge tail risk.

Strategy limitations:
One thing to keep in mind is that to implement the zero-cost collar, an investor's premium paid for the put has to equal the premium received for selling the call, which in reality is very rare, making a perfect zero-cost hedge unlikely. Also, as this strategy caps the gains of a certain investment makes it a short-term hedging strategy, and combined with the fact that the tail risk events are very hard to predict, the timing of the zero-cost collar is very likely to be off.

3.3. Hedging Tail Risk with VIX

Created by the CBOE (Chicago Board Options Exchange), the volatility index (VIX) is a real-time market index that represents the S&P 500's expectation of 30-day forward-looking volatility. In addition, the VIX tends to have, most of the time, a negative correlation with the S&P.

There are multiple ways of hedging a stock portfolio exposure with the VIX index. One could buy VIX futures, buy OTM puts or create a reversed collar using puts and calls. However, as we can see from the graph below, VIX options offer considerably higher returns than the VIX futures.

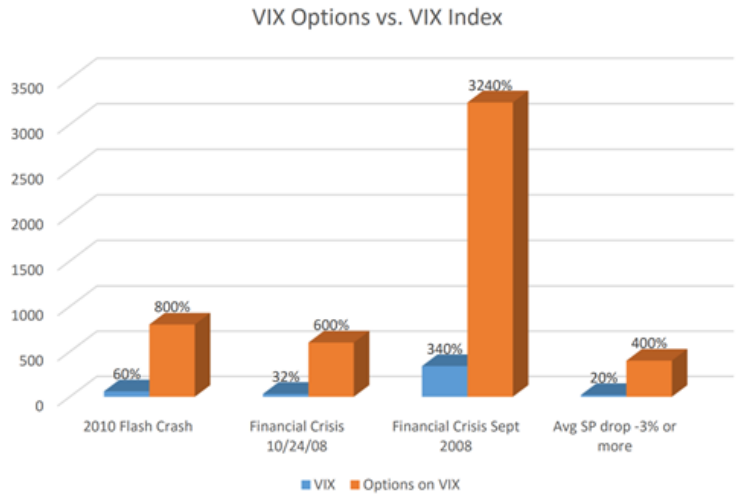


Figure 13. *Return of VIX options vs. VIX futures during past crises*

Reversed Collar VIX
Strategy breakdown:
While buying VIX OTM puts to hedge against tail risk works, it also adds an extra cost that can drive down portfolio performance and tempt the investor trying to find the best market timing to hedge against tail-risk. Yet, given that tail risk is as unpredictable as it is fatal, finding the best timing for such a strategy can be very difficult and risky.

A better approach is to reduce the hedging cost derived from buying OTM puts by implementing a reversed collar VIX options strategy. To do this, an investor needs to buy OTM VIX calls and sell OTM VIX puts for a fraction of his portfolio while keeping the remaining part of the portfolio invested in the S&P 500.

Let us assume the following scenario for implementing a VIX reversed collar:

- An investor has $1,000,000 and it will invest it this way: 100,000 tail-risk hedge + 900,000 SPX
- The S&P 500 Index trades at 1423
- The VIX trades at 16.30
- June VIX calls→ strike price of $19, are priced at $0.40 each
- June VIX puts, with a strike price of $12.50, are priced at $0.25 each

According to the CBOE Website, for every 3% drop in SPX, the VIX rises 16.8% → a 10% drop in SPX should lead to a 56% increase in VIX. For this exercise, let's use the conservative estimate of a 40% rise in VIX when the SPX drops by 10%. Consequently, if the VIX increases 10% from 16.3 to 22.8, each June $19 VIX call will be worth $380 because ($22.8-$19) x $100 = $380. Therefore:

1. Number of VIX calls required to protect 10% of the portfolio is: $100,000/$380 = 264
2. Total cost of purchasing the 264 VIX June $19 calls at $0.40 each = 264 x $0.40 x $100 = $10,560
3. Premium received for selling 264 June $12.50 VIX puts at $0.25 each = 264 x $0.25 x $100 = $6,600
4. Total investment required to construct the hedge = $10,560 - $6,600 = $3,960

S&P 500 Index	VIX	Call Options Value (strike $19)	Put Options Value (strike $12.5)	Net Premium Received (A)	Unhedged Portfolio (B)	Hedged Portfolio (A+B)
711.5 (-50%)	48.9 (+200%)	$789,360	$0	$785,400	$500,000	$1,285,400
1210 (-15%)	26.08 (+60%)	$186,912	$0	$182,952	$850,000	$1,032,952
1280 (-10%)	22.8 (+40%)	$100,320	$0	$96,360	$900,000	$996,360
1352 (-5%)	19.56 (+20%)	$14,784	$0	$10,824	$950,000	$960,824
1423	16.3	$0	$0	($3,960)	$1,000,000	$996,040
1494 (+5%)	13.04 (-20%)	$0	$0	($3,960)	$1,050,000	$1,046,040
1565 (+10%)	9.78 (-40%)	$0	$71,808	($75,768)	$1,100,000	$1,024,232
1636 (+15%)	9.78 (-40%)	$0	$71,808	($75,768)	$1,150,000	$1,074,232
2135 (+50%)	9.78 (-40%)	$0 $0	$71,808	($75,768)	$1,500,000	$1,424,232

***Table 5**. Reversed Collar Performance*

As the ***Reversed Collar Performance Table*** shows, with only a 10% allocation to tail risk via a reversed collar on VIX, the hedged portfolio will perform considerably better than the unhedged portfolio, actually making money during a downturn of 15 or 50%. On the other side, we see that the portfolio upside will be slightly reduced; for example, during a 50% SPX increase, the hedge portfolio will only be 1,424,232 vs. the unhedged portfolio worth 1,500,000.

Strategy benefits:

One of the main benefits of a collar, in general, is that it reduces the hedging cost. Compared with a normal collar, the inverse collar on VIX will cap the upside because, in the case of a roaring bull market, it will only discount from the overall portfolio performance the insurance premium. Also, even if, in theory, the maximum loss of an inverse collar is unlimited, given that historically the VIX has never gone under 9.5.

Strategy limitations:

Historically, the VIX is inversely correlated with SPX only 88% of the time. So, this means there is still a 12% chance that the negative correlation will not materialize. Therefore, using VIX puts or using a reverse collar on VIX will not lead to a 100% perfect hedge.

4. Conclusion

As a final note, we hope that this paper has made clear that tail events - aka Black swan events - are extremely hard to predict, their probability is usually underestimated, and lastly, they have dramatic consequences for the capital markets. As we have also seen, tail risk hedging is a complex and costly strategy, and depending on the timing and strategy's nature, tail risk hedging can either improve or drag down portfolio performance. In fact, the average performance of hedge funds investing only in tail-risk, as represented by the *EurekaHedge index*, shows long-term losses. Hence, one important recommendation would be to allocate a minimum portion of one's portfolio to tail risk hedging. The partial allocation has proven to be a relatively effective tail-risk hedge with a minimum drag for the overall portfolio performance.

Moreover, some option-based strategies have shown positive returns during black swans, without capping the long-term gains. To be more specific, considering our theoretical analysis of different tail risk hedging strategies, the only strategy that in theory should be able to both hedge tail risk and have a minimum cost on the long-term performance would be a 10% allocation to a VIX reversed collar.

Of course, to fully understand the real impact of tail risk hedging for long-term portfolio performance, a mindful investor should take the closest look possible at the secretive world of tail-risk hedge funds. We hope this paper has given to anyone interested in tail risk; some interesting leads to further exploration.

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