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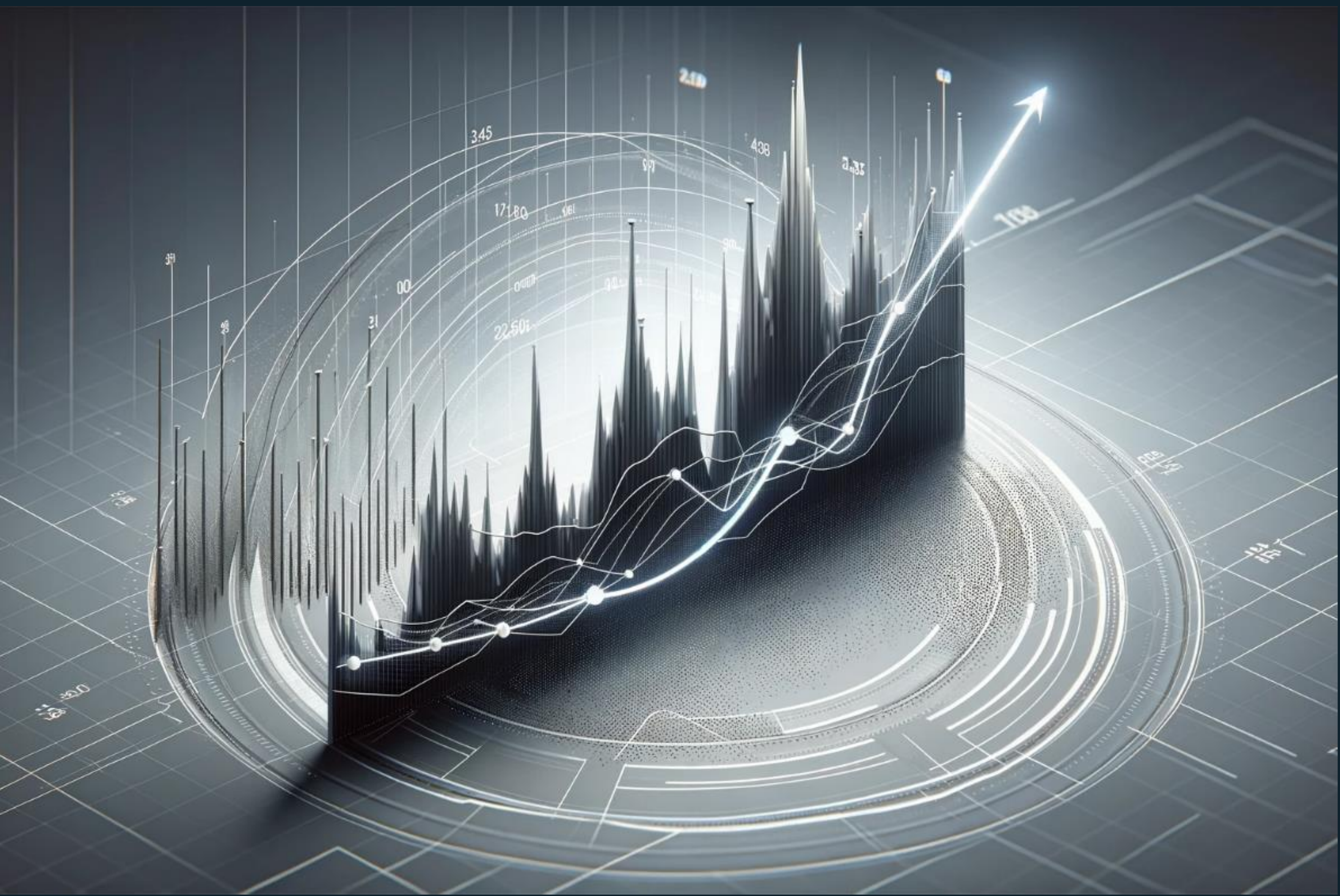
A CASE FOR PATH-DEPENDENT OPTIONS IN AN UNCERTAIN ENVIRONMENT



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Executive Summary

Geopolitical unrest, an impending recession and deglobalization create volatility and demand for specialized solutions that path-dependent options can offer.

The current market environment contains multiple uncertainties, which create volatility and challenges for investors across all asset classes. This report introduces the characteristics and functionality of path-dependent options. This class of options offers crucial opportunities for investors and enables the hedging of portfolios against atypical risks.

We present five popular path-dependent options and explain their behavior and payoffs in diverse scenarios. Moreover, sensibility measures for options are introduced and described to gain an enhanced intuition for the trading of options and expanded knowledge about essential factors influencing options prices. We conduct an analysis to determine how the in options implied volatility behaves and how the implied volatility differs from the volatility observed in the equity market. Moreover, we present fundamental option pricing models for the assessment of an option's fair prices.

The final section asserts the current implied volatility level and discusses whether it is a good time to purchase path-dependent options. Consequently, we provide an overview of decisive events that may impact the market significantly and suggest two path-dependent option strategies to profit from these events.

1 Introduction to Financial Option Contracts

Analyzing the contemporary market volatility is fundamental before introducing options because of their underlying relationship. In recent years, various events have significantly impacted market volatility. The most influential event that altered the markets in the previous decade was the COVID-19 pandemic, which caused a sharp surge in market volatility. The uncertain economic prospects and commercial restrictions led to a significant drop in share prices. Subsequently, the expectations of a recovery due to imminent vaccines led to a partial regain of market losses for the worst-hit industries. However, Russia's invasion of Ukraine introduced high inflation, which affected energy markets and supply chains. Economic nationalism, US-China tensions, and fragmentation have taken root, and governments are taking a bigger role in economic management, particularly when faced with the urgency of the climate transition. The tragic return of conflict to the Middle East only underscores the pattern of rising geopolitical risk. In all these events, the Central Banks raised interest rates to mitigate inflation, but this created more uncertainty and, therefore, more volatility.

In recent years, the market indicators have been showing a trend towards increased uncertainty. This is evidenced, for instance, by implied volatility indices, which measure the expected or implied volatility. They have shown a notable increase in volatility during the outbreak of COVID-19 and have remained on increased levels since. Additionally, the World Uncertainty Index, which tracks the frequency of the word "uncertain" in analysts' reports, has steadily increased for years and has seen a significant jump since 2020 (Ahir, Bloom & Fuceri, 2023). The growing uncertainty about the future is causing market participants, from investors to central bankers, to rethink their strategies. The changing landscape is altering the playbook of market participants, making it challenging to predict the future. As a result, investors are becoming more cautious in their approach, and central bankers are taking a more proactive role in managing the economy.

The volatility in the markets has changed the environment for asset pricing models and increased the difficulty of assessing trends. For over two decades, there was relative stability in the world with growing global trade and few political shocks. This made forecasting macroeconomic variables such as growth, interest rates, and inflation easier since the underlying factors were less volatile and exhibited less noise. However, current economics are increasingly influenced by various factors such as geopolitical situations, increasing raw material and commodity prices, and monetary policies, where an increase in interest rates leads to higher debt costs for companies and slower growth. These factors have significant implications that go beyond the usual scope and influence individuals, corporations, and governments. It is worth noting that markets are not always accurate in pricing geopolitical risks or assessing low-probability, high-impact events, also known as "black swans" in finance. Research indicates that economic activities and financial markets are often more affected by geopolitical threats than actual events. Moreover, when several complex and undefined threats arise, the market can become stagnant and indecisive. For instance, while oil prices rose in response to Hamas' attacks in Israel, they did not increase as much as expected. Therefore, there may be adjustments ahead.

Considering the current volatile and intricate market conditions, options provide a promising alternative for investors seeking new opportunities. An option is a contract between two parties that gives the buyer the right to buy or sell an underlying asset at a certain price within a specific time. The option's value is tied to the underlying asset, which can be stocks, bonds, currencies, interest rates, market indices, exchange-traded funds (ETFs), or futures contracts. Options like stocks or bonds are securities themselves, and they are called derivatives since their value is derived from another asset.

There are mainly two purposes for using options: speculation and hedging. Speculation means investing in options to obtain a profit from the price movement of the underlying, usually focusing on volatile prices or events. This can be achieved through several trading strategies, such as buying call options to benefit from an increase in stock price or buying put options to profit from a decline. The primary tool used by speculators is leverage, which allows them to amplify potential gains or losses with a smaller upfront investment. On the other hand, options can also be used for hedging purposes, serving similar to an insurance policy to cover risks. Investors can use options to protect their investments from downside risk, especially during periods of high market volatility.

For instance, buying put options can protect a stock portfolio from a market downturn, while buying call options can protect a short position from a sudden price increase. Hedging with options is also used to lock in a specific price for an asset, such as a commodity, to reduce uncertainty and allow for better financial planning. Ultimately, the use of options requires a good understanding of market dynamics and risk management strategies to achieve the desired financial outcomes.

Furthermore, there are diverse reasons why people utilize options. One of the applications is to provide capital protection in structured products by using options to minimize potential losses. Call options are used to limit the risk of potential losses and are a risk management tool to define risk levels in a portfolio. This is especially useful for investors who want to protect their investments while still being able to benefit from market movements or traders who want to manage their risk exposure and maintain a balanced portfolio. Aside from capital protection, options can also be used for income generation. The issuance of options, called ”writing” options, can generate premiums and additional income from existing assets, which can be a valuable source of additional income for issuers. Traders can also exploit price disparities between options and their underlying assets for risk-free profits, known as arbitrage trading. Volatility trading is another way traders can profit from options, irrespective of market direction. Options can also be used to mimic the risk-return profiles of assets, offering greater flexibility in trading strategies. This is particularly useful for investors who want to diversify their portfolios and take advantage of different market conditions. Lastly, companies can use options, such as employee stock options, as a form of compensation and capital raising.

Options are widely utilized by various market participants for different purposes. For individual investors, options are primarily used to generate income, hedge against price movements in their existing portfolios, and speculate on the future price movements of assets. Institutional investors with substantial capital use options to hedge their portfolios against adverse market movements. They employ diverse strategies, such as buying puts or writing calls, to protect their portfolios from potential losses. On the other hand, hedge funds utilize complex strategies involving options and other financial instruments to generate returns for their investors. They use options for strategic investments, hedging against risks, and speculating on price movements. This allows them to hold a leveraged position in an asset at a lower cost than buying shares of the asset. Moreover, traders can use options to implement numerous trading strategies, such as straddles, strangles, and spreads, which provide them with a wider range of investment and profit opportunities.

Options come in many different forms, while the most common types are equity-based options with stocks as underlying. The buyer of a call option receives the right but not the obligation to purchase the underlying stock at a certain price. Similarly, the investor of a put option receives the right but not the obligation to sell the underlying at the determined price. The advantage is that the option buyer is not required to own the underlying security. Besides individual stocks, investors can also buy options based on a stock index, which is an artificial basket comprising numerous different stocks meeting certain criteria. Stock indices are usually created so that investors and economists can monitor the overall market quickly and easily. Moreover, traders have the option to invest in commodity options and foreign exchange options. In commodities options, the underlying asset is a commodity such as gold, crude oil, or agricultural products. In contrast, foreign exchange options have currencies and their respective exchange rates as their underlying. Investors can use these options to enter positions on the future value of the underlying assets and potentially earn profits. Overall, these different options provide traders with a diverse range of investment opportunities to explore.

1.1 General Characteristics of Options

As mentioned before, options are a type of financial derivative that grants the purchaser the right but not the obligation to either sell or buy a security or other financial asset. This type of contract includes an agreed-upon price and a specified period or date in which the investor can exercise the option contract to receive the underlying asset. Conversely, if the option holder chooses to exercise it, the seller of the option is required to deliver the underlying. The two parties involved in an option contract are the option buyer or holder and the option seller or writer. When the buyer decides to initiate the transaction, they are said to be exercising the option, whereas if the option contract is not exercised before or at maturity, the option expires worthless.

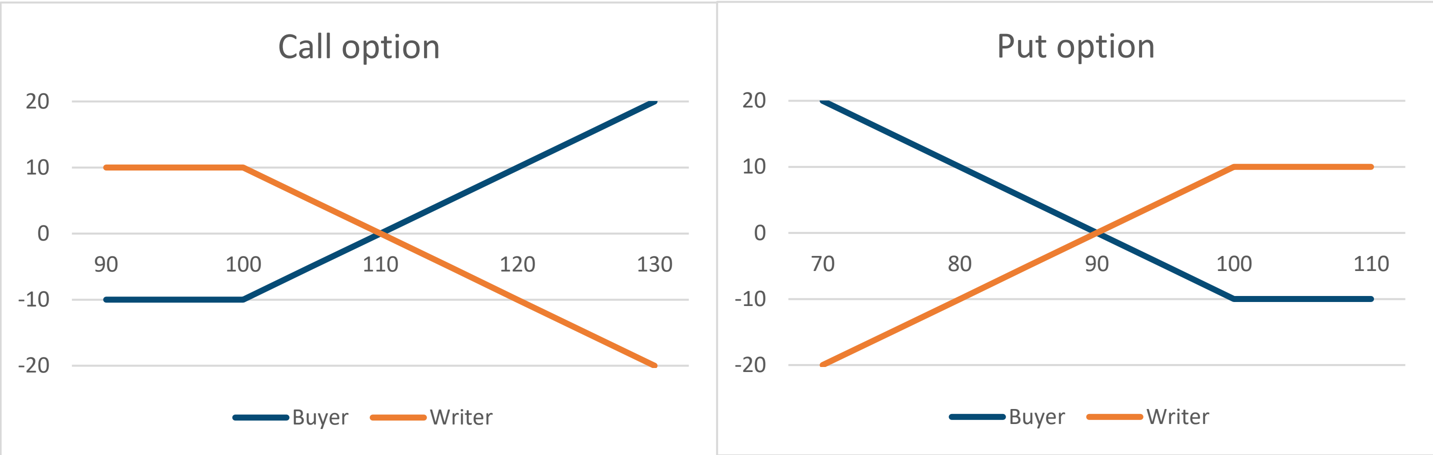
When dealing with options contracts, it is crucial to understand some of the main terminologies. Firstly, having a call option means that the holder has the right to purchase an underlying asset on a predetermined date or period at a specified price. Conversely, a put option gives the holder the right to sell an underlying asset at a predetermined date or period and a specified price. An European option can only be exercised at the specified maturity date, while an American option can be exercised at every arbitrary point in time until the option expires. Investors who purchase call or put options are said to be in a long position since they have the option to buy or sell the underlying asset at the strike price. The execution of an option means that the investor believes that the option payoff will not increase in the future. A call option pays out the difference between the current price of the underlying minus the strike price. Thus, the investor is assuming the underlying to appreciate in value. Conversely, a put option pays out the difference between the strike price and the current price of the underlying. Thus, the investor is assuming the underlying will depreciate.

The strike price is the price at which a contract can be executed and the price at which the underlying asset can be bought or sold. When an option's underlying asset price is equal to its strike price, it is known as being "at the money". When the underlying asset moves in the intended direction and the payoff of the option increases, then the option is “in-the-money”. Otherwise, the option is “out-of-the-money”.

The contract size refers to the number of underlying assets that are covered by the option contract. For example, in the stock options market, the typical contract size is 100 shares. This means that when an investor purchases one options contract, they are essentially buying the right to either buy or sell 100 shares of the underlying stock at a predetermined price within a specific time frame. Understanding the contract size is crucial when it comes to trading options, as it can impact the overall cost and potential profits.

The option’s premium is the price that needs to be paid to acquire the options. The premium comprises two main components: intrinsic value and time value. The intrinsic value is the difference between the underlying price and the strike price. Therefore, for call options, the intrinsic value is the current stock price minus the strike price, while for put options, it is the strike price minus the current stock price. The time value is the amount paid for the residual time for the market to move in your favor. The premium equals the intrinsic value plus the time value. Various factors can affect the options premium, such as higher prices resulting in higher premiums for call options and lower premiums for put options, the selection of the strike price affecting the possibility of the option being in-the-money, and volatility increasing the likelihood of the option being in the money.

Now, we'll discuss the payoffs of plain vanilla options after presenting some basic knowledge. When someone buys a call option, they pay the option premium at the time of entering the contract. This means that the buyer can potentially make a profit if the market moves in their favor. The most attractive feature of buying options is that the maximum loss is limited to the option premium paid. With a limited investment, the buyer secures unlimited profit potential with a known and limited potential loss. However, if the spot price of the underlying asset does not exceed the option strike price before the option's expiration date, then the investor loses the amount paid for the option premium, equaling a total loss. On the contrary, if the price of the underlying asset exceeds the strike price, then the call buyer makes a profit, which is the difference between the market price and the option's strike price minus the price paid for the option. For example, in the graph on the left below, if a trader buys one call option contract on Company A stock with a strike price of USD 100, paying USD 10 for the option and on the option's expiration date, Company A’s shares are selling for USD 120, the buyer exercises their right to purchase 100 shares at the strike price of USD 100 a share and immediately sells the shares at the current market price of USD 120 per share. The profit is USD 120 – (USD 100 + USD 10) = USD 10. If the buyer bought one option contract, their profit equals USD 10 x 100 shares = USD 1,000. Thus, their net profit, excluding transaction costs, is USD 1,000, which is a significant return on a USD 1,000 investment.



On the other hand, if someone sells a call option, their downside is potentially unlimited. If the spot price of the underlying exceeds the strike price, then the writer of the option consequently incurs a loss, which is equal to the profit of the option buyer. However, if the market price of the underlying is below the option strike price, then the option expires worthless. The option seller profits by the amount of the premium they received for the option.

A put option gives the buyer the right to sell the underlying asset at the strike price. The profit the investor receives on the option depends on how far the spot price falls below the strike price. If the spot price is lower than the strike, then the buyer of the put option is "in-the-money." Conversely, if the spot price remains above the strike, then the option will expire worthless and unexercised. The writer of the put option suffers a loss if the spot price of the underlying is lower than the strike of the contract. The loss is equal to the payout of the option’s buyer minus the option’s premium. The option expires unexercised if the spot price remains above the strike of the contract and the writer receives the full option premium. The graph above shows the profit or loss for a hypothetical call and put option, with an option premium of USD 10 and a strike price of USD 100. The potential loss for the buyer is limited to the cost of the put option (USD 10). The seller, or writer, of the put option receives a profit as long as the price of the stock remains above USD 90.

2 Characteristics of Path-dependent Options

2.1 Exotic Options

One category of options is standardized option contracts that have regulated terms and conditions established by the options exchange on which they are traded. These standardized features help to create uniformity and transparency in the options market, making it easier for investors to understand and trade these financial instruments. On the other hand, the second category of options are non-standardized options, also called exotic options, that have been gaining popularity among different investors and are now being listed on public exchanges. Unlike standardized options, exotic options are customized contracts that have complex payoff structures, underlying assets, or expiration conditions. Exotic contracts are not traded at options exchanges such as the Chicago Board Options Exchange (CBOE) or the Eurex but instead are traded in the over-the-counter (OTC) market, where the issuers sell exotic options directly to the investor, making them not as easily accessible and liquid as standardized options.

The best-known exotic options include Barrier options, Lookback options, Chooser options, Asian options, Binary options, Compound options, Basket options, and Bermuda options. For example, Binary options are contracts that pay out a fixed amount of money when the option fulfills a certain criterion, for instance, if the underlying stock reaches a certain price. The payout follows a simple "yes" or "no" proposition where the determined amount is paid out if the criterion is satisfied, or otherwise, nothing is paid out. Binary options have maturities varying from weeks or months down to mere minutes or even seconds. Compound options are options where the underlying security is another options contract. These options are often used in situations where the underlying security is difficult to value or when the investor wants to hedge against potential losses. Basket options allow investors to trade a group of assets as a single unit, which can be useful for diversification purposes. Bermuda options can be exercised at specific dates before the expiration date, providing investors with more flexibility.

2.2 Path-dependent Options

Path-dependent options are exotic options whose value depends not only on the price of the underlying at maturity but also on the path of the underlying between the issuance of the option contract and its maturity.

Compared to traditional options, path-dependent options tend to be more sophisticated and, therefore, harder to price. This often translates into adjustments made to classic option pricing models such as the Black-Scholes model. These limitations gave the incentive to create new elaborate option pricing models to price exotic options such as path-dependent options. The reason is that the Black-Scholes model delivers an accurate approximation for pricing European options, but it falls short when the entire price trajectory becomes relevant. As new models and ways to look at derivatives pricing were being created, new derivatives had a possibility to be created. One example of this is precisely path-dependent options, in all their shapes, which allowed investors to create, for the first time, more personalized trading and hedging strategies that could exploit the entire journey of the asset's underlying and not just its end price. One commonly used approach to valuing path-dependent options is the Monte Carlo simulation, which simulates numerous trajectories whose final prices are discounted to yield the expected price.

The adaptability of these options makes them crucial to manage complex risk management and investing strategies. During this chapter, we will see how different types of path-dependent options can be applied to help customize investment strategies in specific scenarios. Consequently, five path-dependent options will be subsequently introduced, and their characteristics and behavior will be explained.

2.3 Important Path-dependent Options

2.3.1 Barrier Options

A type of exotic option in which the payout depends on whether the underlying asset has reached or exceeded a predetermined level is called the barrier (B).

Option Types

Generally, it is differentiated between four main types of barrier options based on the combination of the following two characteristics.

1. *Knock-In vs Knock-Out*: A knock-in barrier option is triggered if the underlying asset's price reaches the barrier level. In other words, before reaching the barrier, the option is "dormant"; it is almost like no contract was present between the counterparts, and the intrinsic value is zero. When the barrier is triggered before the maturity date, then the barrier option is activated and converted into a plain vanilla option. A knock-out option, on the contrary, loses its value and ceases to exist if the underlying hits the barrier during the life of the option.
2. *Up vs Down*: The barrier level can either be higher ($B > S_0$) for a call option or lower ($B < S_0$) for a put option than the value of the underlying at the beginning of the contract period (S_0).

Combining these two characteristics, we get four possible types of barrier options:

- *Up-and-Out*: They expire worthless if, during the life of the option, the underlying asset's price exceeds the barrier level, which was initially set higher than S_0;
- *Down-and-Out*: They expire worthless if, during the life of the option, the underlying asset's price falls below the barrier level, which was initially set lower than S_0;
- *Up-and-In*: They are activated if, during the life of the option, the underlying asset's price exceeds the barrier level, which was initially set higher than S_0;
- *Down-and-In*: They are activated if, during the life of the option, the underlying asset's price falls below the barrier level, which was initially set lower than S_0.

Each of the mentioned types of options can be either a call or put option.

Payoff Functions

Payoff of an Up-and-In call:

$$\text{payoff} = \begin{cases} \max(S_T - K, 0) & if\ S_T \geq B\ before\ maturity \\ 0 & if\ S_T < B\ until\ expiration \end{cases}$$

with S_T being the option's payoff at maturity.

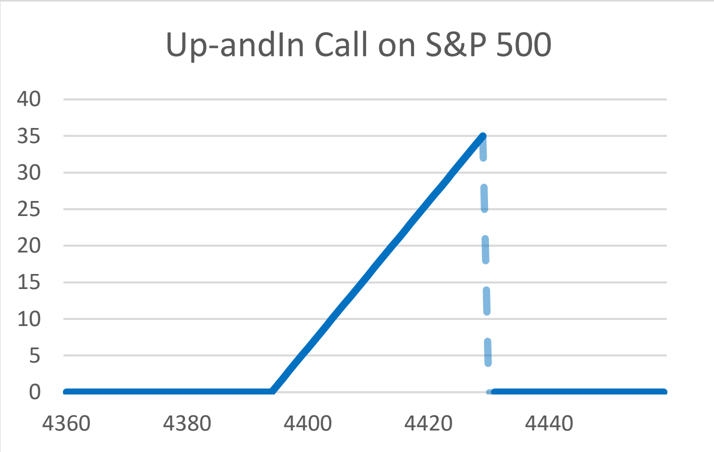
Payoff of an Up-and-Out call:

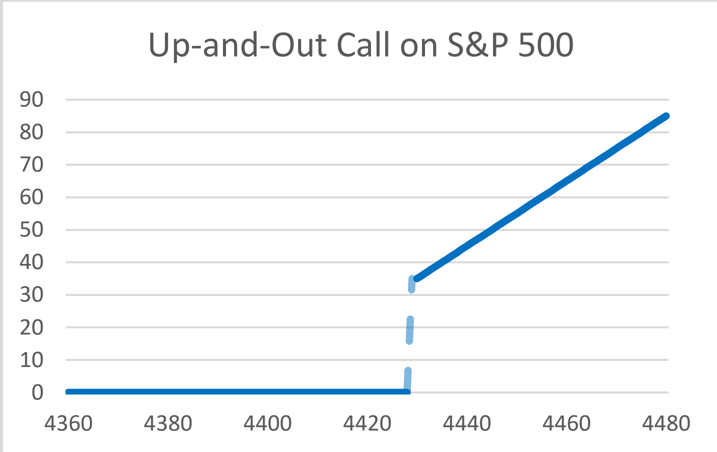
$$\text{payoff} = \begin{cases} 0 & if\ S_T \geq B\ before\ maturity \\ \max(S_T - K, 0) & if\ S_T < B\ until\ expiration \end{cases}$$

The payoff functions of the equivalent put options can be obtained by changing $\max(S_T - K, 0)$ in both equations to $\max(K - S_T, 0)$. The payoff functions of the equivalent call and put options with B lower than S_0 can be obtained by changing the $\geq$ with a $\leq$.

Payoff Diagrams

The payoff diagrams of an Up-and-In call option on the S&P500 with a barrier at B= 4430 and a strike price at K = 4395 and an Up-and-Out call option on the S&P500 with a barrier at B= 4430 and a strike price at K = 4395 are given below.





Rationale of Barrier Options

But why would anyone want to buy an option where part of the profit is "left on the table"? This potential loss in profit can be seen if we compare both presented graphs with that of a plain vanilla call option with the same strike price. In both cases, there is a range of values for the terminal payoff, which is equal to zero for a barrier option and non-zero for the equivalent plain vanilla call option. In other words, there is an extra condition that needs to be satisfied to make the option profitable. It is not enough for the payoff at maturity to be higher (calls) or lower (puts) than strike K at maturity to have a non-zero payoff. The answer can be found in barrier options' premiums. Because of their underlying mechanics, barrier options cost less than plain vanilla options on the same underlying, with the same strike K and expiry date. It is worth noting that at maturity, the payoff of a knock-in plus the payoff of a knock-out option equals the payoff of a standard option.

This means that if, for example, an investor anticipates very high volatility on the underlying and has bullish expectations, he might buy an up-and-in call, which will reduce the cost of such an operation compared to simply buying a call option. Not only do they reduce costs, but they might be more suited to particular expectations. If, for example, a trader thinks that a stock will bounce back only if it first breaks a support level, he may buy a down-and-in call option with a barrier on the support level or close to it. We make one last example. Let's suppose an investor wants to hedge a long position in a stock and thinks that if the stock reaches a certain peak value Z, then it will crash, he might do so by buying an up-and-in put option. Such a position means the following: "I don't think I will need the protection of a put on the stock I own unless the stock price reaches a certain value Z, if that's the case, I believe it may crash and consequently would want my put option to hedge my stock."

General Information

An important issue for barrier options is the frequency with which the asset price (S) is observed for the purposes of determining whether the barrier has been reached (Hull & Barone, 2018). Obviously, the higher the frequency, the more likely the barrier will be reached. This means that an increase in observing frequency will make knock-in options more expensive. The opposite is true for knock-out options. Sometimes, the underlying asset is observed continuously, which is also the typical assumption made in pricing these derivatives.

It's worth noting that traditional barrier options can be sensitive to sudden and brief fluctuations in the asset's price. These temporary spikes can unexpectedly activate the knock-in or knock-out conditions, potentially leading to unintended consequences for the option holder. This is the reason for the existence of Parisian options. Parisian options require the asset price to exceed or drop below the barrier for a set period before the option's conditions are triggered. This approach guards against the option being affected by short-lived price movements, providing a more stable and measured response to market changes.

2.3.2 Lookback Options

Lookback options are path-dependent options in which the payoff depends on the maximum (call) or minimum (put) price of the underlying asset reached during the option's life. In other words, lookback options allow the holder to capture the maximum payoff of the most favorable time as if the holder was going back in time and exercising at the most convenient moment.

Option Types

Lookback options can be distinguished into two categories:

a) *Fixed lookback options*: The strike price K is fixed. What the buyer chooses at maturity is the most convenient underlying price. In the case of a call, the holder will receive the maximum price reached in the option's life, in the case of a put, he will receive the minimum. Therefore, the payoff of a fixed lookback call/put option is the same as a regular European call/put, except that the final asset price is replaced by the maximum/minimum price achieved during the life of the option.

b) *Floating lookback options*: There is no defined strike price, but as the strike is chosen for the holder, the most convenient value between all the values the underlying has reached throughout the life of the option. This means that the payoff of a floating lookback call is the amount that the final asset price exceeds the minimum asset price achieved during the life of the option [...], whereas the payoff from a floating lookback put is the amount by which the maximum asset price achieved during the life of the option exceeds the final asset price (Hull & Barone, 2018).

Payoff Functions

Fixed Lookback Call:

$$\text{payoff} = \max\left(\max(S) - K, 0\right)$$

Fixed Lookback Put:

$$\text{payoff} = \max\left(K - \min(S), 0\right)$$

Floating Lookback Call:

$$\text{payoff} = \max(S_T - min(S), 0)$$

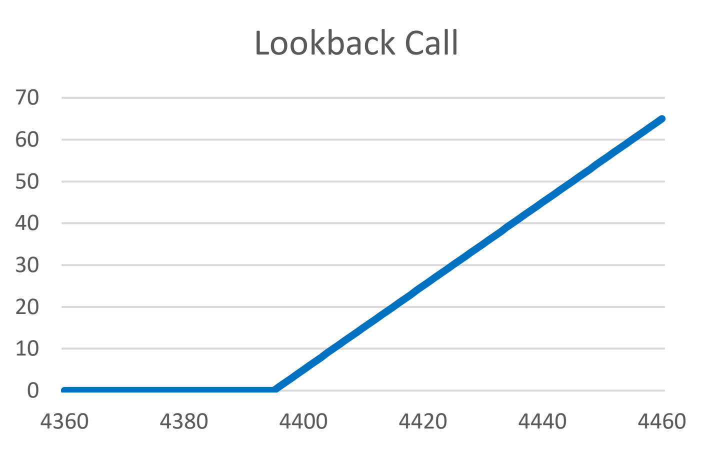
Floating Lookback Put:

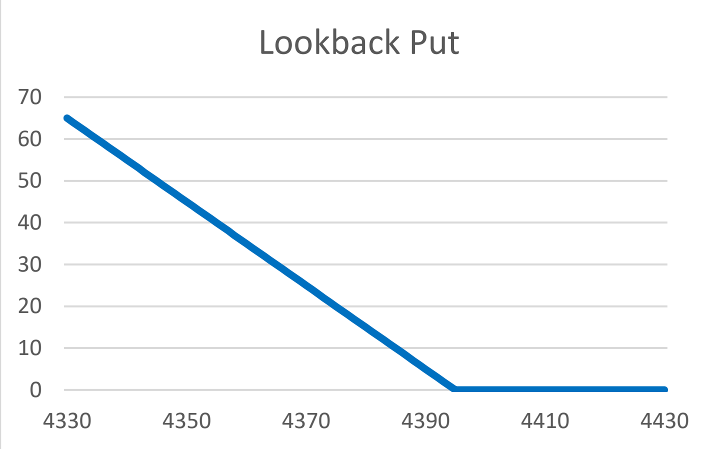
$$\text{payoff} = \max(max(S) - S_T, 0)$$

By min(S) and max(S), we respectively refer to the minimum and maximum price the underlying asset has reached during the option's life. It is worth noting that lookback options cannot yield a negative payoff. The very unlikely worst case for the buyer of a lookback option is that the price of the underlying does not change and is identical until the expiration date. In the case of a floating lookback option, the payoff would be zero. Conversely, the payoff is positive as soon as the price of the underlying fluctuates.

Payoff Diagram

The payoff diagram of a fixed lookback call looks similar to the payoff diagram from a plain vanilla European option, with the only difference being that the variable on the x-axis is the maximum price reached for a call option and the minimum price reached for a put option.





Rationale of Lookback Options

Lookbacks are clearly appealing based on their beneficial payoff structure. A floating lookback call is a way for the buyer of the option to buy the underlying at the lowest price during the life of the option, whereas a floating lookback put allows holders to sell at the highest. This benefit obviously has a cost that translates into higher premiums, as is always the case in finance: there is no free lunch. As with barrier options, the value of a lookback option is liable to be sensitive to the frequency with which the asset price is observed for the purposes of computing the maximum or minimum.

2.3.3 Russian Options

Russian options, also known as perpetual lookback options, are lookback options with one particular characteristic: no expiration date. This means the holder can hold them indefinitely and exercise them at the most convenient underlying price ever recorded since the inception of the contract. There are no traded Russian options on the public option exchanges because such an advantageous position of the holder would translate into very expensive premiums that would also be quite hard to quantify. These options are only traded OTC, this is because they would be hard to standardize and possess crucial customization. They are very complex contracts, hard to price, and would also most likely not create enough liquidity to be exchange-traded. Moreover, not having an expiration date, therefore allowing for the continuation of a derivative contract in the long term, would dramatically increase counterparty risk. All these characteristics could make Russian options less interesting than they might seem at first sight.

Payoff Function

The payoff function of Russian call options is identical to the lookback options.

2.3.4 Chooser Options

A type of exotic option in which the holder, after a specified period of time, can choose whether it is a call or a put. This means that the value of these types of options at the time of choice is the following:

$$\max(c, p)$$

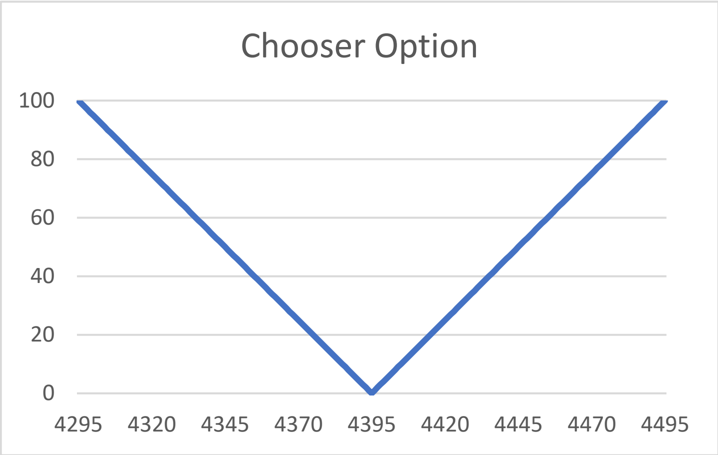
Where c and p respectively indicate the values of a plain vanilla European call and put option. There can be more complex chooser options characterized by the fact that the call and put have different strike prices or times to maturity.

Payoff Function

Once chosen, the holder of the option will receive either a call or a put option, which means that the payoff at maturity will be a plain vanilla European option (either call or put) with the specified terms. The only difference with a plain vanilla option is that this payoff function becomes apparent only after the time of choice, depending on which of the two was more convenient at that time, not at the inception of the contract.

Assuming the investor always chooses the more favorable option, the payoff curve at maturity is given in the subsequent graph. The payoff of the chooser option is comparable to a straddle, which is a trading strategy that consists of buying a European call and put with the same expiration date and strike prices. The payoff function is given by

$$\text{payoff} = \max(c, p, 0)$$



General Information

The price of a chooser option is more complex than plain vanilla options and is, in general, more expensive. This increase in the premium is due to the increased level of flexibility, which allows the investor to have "an option on an option". You first have the option to choose between put and call and receive the usual payoff from the chosen option.

Chooser options can have great applications in corporate finance. Suppose a company needs an amount X of copper in 9 months, but there is high uncertainty on the price of this commodity. The CFO could use futures to hedge against this risk. However, there are certain conditions under which the CFO may prefer a chooser option with a decision date that is between the present and nine months in the future:

- If there is uncertainty about the direction, high volatility is anticipated. In this case, futures could lock the company into a deal that could result in an unfavorable position, whereas the option would keep the company's options open until the moment of choice. Although the CFO has to pay the premium for the chooser option to the issuer, a sure cost amount today might be preferred by the CFO. In general, chooser options can be more cost-effective in certain situations despite the enhanced premium.
- Options allow for a higher leverage than futures. The premium initially paid to enter into an option contract is usually lower than the initial and maintenance margins required by the clearing house to keep a future position open.
- Being traded OTC rather than on an exchange, chooser options allow for more personalization, which can be very useful in corporate finance.

2.3.5 Asian Options

A type of exotic option that can only be exercised at maturity, where the final payoff depends on the average price of the underlying asset during the life of the option. Depending on the type of Asian option used, both the computation of the average and, consequently, its final payoff will change.

Types and Payoff Functions

First of all, we can distinguish between ”average strike” and ”average price” Asian options.

- *Average strike*: The strike to be used to calculate the payoff is equal to the mean of the underlying price throughout the life of the option. The calculation of the mean is used to randomize the strike price at which the holder will be allowed to exercise at maturity. The payoff functions of an Asian average strike call and put option with average strike (K_T) are:

$$\text{call payoff} = \max\,(S_T - K_T; 0)$$
$$\text{put payoff} = \max\,(K_T - S_T; 0)$$

- *Average price*: The price at maturity used in European options is replaced by the average of the price throughout the life of the option. The consequent payoff functions are:

$$\text{call payoff} = \max\,(M_T - K; 0)$$
$$\text{put payoff} = \max\,(K - M_T; 0)$$

Notice that M_T is always the average of the underlying asset's price. Therefore, it is always calculated in the same way, independent of whether it is in the payoff function of an average price or average strike option.

However, based on how we calculate the average, we can make an additional distinction:

- In the case of an ”arithmetic mean”, the formula for M_T would be:

$$\frac{1}{n}\sum_{j=1}^{n} S_j$$

- In the case of a ”geometric mean”:

$$\left(\prod_{j=1}^{n} S_j\right)^{\frac{1}{n}}$$

General Information

Asian options are less volatile than European plain vanilla options. This is due to the smoothing effect of calculating the mean, which diminishes the effects of extreme movements in the underlying asset price. As previously explained, more volatility in the final payoff is positive for the holder of an option because of the asymmetric nature of such instruments, therefore making Asian options, on average, less expensive than equivalent plain vanilla options.

Asian options can be applied in numerous ways and are especially useful because of their ability to be less affected by a volatile underlying. One example would be a US company that is expected to receive cash flows from European clients at different dates in a certain year. One cost-effective possibility of dealing with fluctuations in the EUR/USD exchange rate throughout the year would be to buy an average price Asian put options on EUR/USD. The option would be a put because the idea of the hedge would be that of protecting the company's inflows from a weakening of the euro, which would be valued less in dollars. This position would translate into the possibility of mitigating the loss due to a less favorable exchange rate over the period in which the cash flows are expected (which would happen if the average exchange rate were to be below the strike price of the option).

3 Key Option Sensitivities and Volatility

3.1 The Greeks as Risk Measures

The Greeks are a collection of partial derivatives employed in derivatives pricing to measure the sensitivity of option prices to shifts in single variables that an investment strategy comprising options is contingent on. The Greeks, as risk measures, constitute a central element for efficient trading strategies, and an in-depth understanding of them is crucial for portfolio hedging and rebalancing of portfolios.

In this section, we will delve into seven Greeks whose comprehension we deem valuable to the reader approaching the intricate discipline of options trading. The computation of the following sensitivities is carried out in line with the Black-Scholes formula for illustration purposes despite some shortcomings in the resulting implied volatility (Haugh M., 2016).

Table 1: The matrix of Greeks

Option parameter	Spot price (S)	Volatility (σ)	Passage of time	Interest rate (r)
Theoretical option value (V)	Delta (Δ)	Vega ($\mathcal{V}$)	Theta (Θ)	Rho (ρ)
Delta (Δ)	Gamma (Γ)	Vanna	Charm	-

Delta (Δ) measures the change of the option's premium associated with swings in the price of the underlying (Ahn J. et al., 2012) and is computed as the first derivative of the theoretical option value V in regard to the price of the underlying security S.

The delta of a European call option with premium C is

$$\Delta_{call} = \frac{\partial C}{\partial S} = e^{-q\tau}\Phi(d_1)$$

Where q is the continuous dividend yield, $\tau = T - t$ is the contract's residual time to maturity, $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution $d_1 = \frac{\ln\left(\frac{S_t}{K}\right)+\left(r-q+\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$, σ is the standard deviation of the stock's log price process and serves as a measure of volatility, and K is the exercise price.

According to the put-call parity, which defines the relation of call options to put options, the delta of a European call option with premium P is

$$\Delta_{put} = \partial P/\partial S = \Delta_{call} - e^{-q\tau} = e^{-q\tau} \cdot [\Phi(d_1) - 1]\,.$$

The value of delta ranges from -1.0 to 0.0 for long put options because an increase in the underlying price leads to a decrease in the option's premium and from 0.0 to 1.0 for long calls since a decrease in the underlying price leads to a decrease in the option's premium.

It is noteworthy to point out that the premium of an option contract behaves progressively more like the price of the underlying instrument as the option premium increases and becomes more in the money (ITM), with delta converging on 1.0 for a call and -1.0 for a put.

The absolute value of delta is approximately equal to the probability that the contract will expire in the money.

If we assume that the Black-Scholes model with log returns holds, then the delta is approximately equal to the probability that the contract will expire in the money.

Trading advice: We label as “delta neutral” a portfolio comprised of options and their respective underlying securities, where both are traded in a manner such that positive and negative components of delta offset each other, thus totaling the overall delta of the strategy to zero and leading to the value of the portfolio becoming reasonably insensitive to fluctuations in the prices of the underlying assets for a certain range of values. Delta-neutral strategies can be implemented for hedging purposes. If you prevent the delta of a portfolio from shifting away from zero, you can effectively “lock in” the future payoff of the options (Hull & White, 2017). Nevertheless, in reality, this form of hedging produces considerable transaction costs. Hence, portfolio rebalancing is carried out intermittently, e.g., every three months.

Gamma (Γ) quantifies the rate of change of delta measured against the rate of change of the underlying asset’s price, and it is calculated as follows:

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = e^{-q\tau} \frac{\phi(d_1)}{\sigma S \sqrt{\tau}}$$

Where $\phi(\cdot)$ is the standard normal probability distribution function (PDF).

Gamma is maximized when the option is at the money (ATM), and it shrinks the further the option deviates from its strike price.

Trading advice: When setting up a delta hedge for your strategy, you can additionally protect yourself from a broader array of price oscillations of the underlying asset by neutralizing the gamma of the portfolio. You can achieve this by introducing additional option contracts to your existing strategy that have offsetting gamma values, analogous to what we explained for delta-neutral portfolios. Likewise, the investor should consider transaction costs for adjustments of the hedging position.

Vega ($\mathcal{V}$) measures the sensitivity of the option price to a change in the underlier’s volatility, and it is computed as

$$\mathcal{V} = \frac{\partial C}{\partial \sigma} = e^{-q\tau} \cdot S\sqrt{\tau}\, \phi(d_1).$$

Such risk measure expresses the increase (decrease) in the theoretical value of the option as the standard deviation (σ) of the underlying instrument’s log returns rises (shrinks) by 1 percent point, independently of the price of the underlier.

Market participants who are holding a long call or long put option can profit from a surge in volatility, as it amplifies the probability that the contract will end up in the money, thus increasing the value of the option that they hold.

For a longer time left to expiration, an option's premium is influenced by volatility to a greater extent. As a consequence, Vega decreases as the expiry date of the contract approaches because volatility has less time to increase the option’s premium.

Trading advice: If you have confidence in your forecasts of higher future volatility of the underlying security, you can implement a strategy with contained risk named “long straddle”. It consists of buying a call option and a put option with the same expiration date and exercise price. You will benefit from profound swings in the price of the underlying in either direction as long as it shifts far away from the strike. While there is unlimited profit potential, the loss is constrained to the premia paid for the two options.

Vanna is the second-order derivative of the theoretical option value with respect to the spot price and volatility of the underlier, and it is computed as follows:

$$Vanna = \frac{\partial^2 V}{\partial S\, \partial \sigma} = \frac{\partial \Delta}{\partial \sigma}.$$

Due to its mathematical equivalence with the sensitivity of the option delta to a shift in the standard deviation of log returns, this risk measure can help predict variations in the efficacy of a delta hedge as volatility varies. For this reason, Vanna is also denoted as "DdeltaDvol".

Theta (Θ) is a measure of how time decay affects the option premium (Ahn J. et al., 2012), and it is the first derivative of the option price to a negative change in time-to-maturity (Haugh, M., 2016):

$$\Theta = -\frac{\partial C}{\partial T} = -\, e^{-q\tau}\, S\phi(d_1)\frac{\sigma}{2\sqrt{\tau}} + qe^{-q\tau}\, S\Phi(d_1) - rKe^{-r\tau}\Phi(d_2)$$

Where r is continuously compounded risk-free rate and $d_2 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$.

Generally, the extrinsic value of the option decreases over time. Indeed, the value of an option diminishes with time due to an increasingly lower likelihood of realizing a profit. Thus, options always have a negative Theta since an option loses some value every day.

Rho (ρ) quantifies the sensitivity of the theoretical value of the option to the annualized riskless interest rate prevailing on the market and is expressed as the monetary amount an option will lose or gain with a 100bps change in interest rates:

$$\rho = \partial V / \partial r.$$

Even though interest rate risk is marginally considered in contemporary trading strategies in an overall stable interest rate environment, a trader could take into consideration the cost of carrying a strategy over time rather than, for instance, investing cash in the money market. Contracts with a longer time to expiration and a higher initial price for the underlying asset are more sensitive to shifts in risk-free rates.

3.2 The Concept of Volatility

We reckon that expanding on the concept of volatility is an informative prerequisite to understanding the relation between historical and implied volatility, which we will develop in the subsequent section.

In financial econometrics, the term "volatility" generally refers to one of two components featured in a lognormal random walk model, which represents the risk of a change in a securities price, and it is measured as the standard deviation (σ) of log returns $\{r^*{}_t\}_{t=1}^{T}$ for the price series of a traded security. The standard deviation of a random variable, a statistical population, or a sample is, by definition, the square root of its variance $V[\cdot]$, therefore:

$$\sigma = \sqrt{V[r^*{}_t]}$$

The variance is the expected value of the squared deviation from the mean of a random variable. To calculate the volatility of a security underlying an option contract, we need to estimate its variance of returns through an unbiased estimator:

$$\mathbb{E}[\hat{\sigma}^2] := \mathbb{E}\left[\frac{1}{n}\sum_{i=0}^{n}(r^*{}_{t-i} - \bar{r})^2\right] = \sigma^2$$

Where n denotes the number of observations and $\bar{r}$ is the mean of the log returns.

Volatility is a synonym of uncertainty and risk in finance, as it pertains to the magnitude of price swings for securities. Higher volatility implies that the price of an asset can fluctuate over a wider range of values, entailing less predictability and, thus, higher risk. It is worth mentioning that volatility is not only a measure of downward risk. In fact, high volatility can turn out to be an investor's ally under certain circumstances. For instance, a rise in volatility will generally benefit the holder of an option, as it will amplify the probability that the contract will end up in the money.

It is essential to differentiate between historical volatility and implied volatility.

Historical volatility *("HV")* is calculated by taking the average standard deviation of returns for a given historical period (Egly & Solbakke, 2022). Daily time-varying volatility models where volatility is explicitly modeled as the second moment of daily returns are called historical volatility models and they include stochastic volatility *("SV")* and generalized autoregressive conditional heteroskedasticity *("GARCH")* classes of models. Historical-based methods are, by definition, backward-looking, as they are calculated from observed past prices of securities (Koopman, Jungbacker & Hol, 2005).

The **implied volatility** *("IV")* of an option is the one entailed by the current market price of the option based on a pricing model. Hence, implied volatility is a forward-looking measure, unlike historical volatility, which is calculated from observed past prices of securities (Ahn J. et al., 2012). The Black-Scholes model estimates a theoretical value (V) with a set of input parameters, among which is an estimate of the future realized volatility (σ) of the underlying asset. Mathematically, for a pricing model $f(\cdot)$:

$$V = f(\sigma, \quad \cdot).$$

The function f is monotonically increasing with σ, thus denoting that higher volatility leads to a higher theoretical value of the option, in line with the notions discussed when we presented the risk measure, Vega. Taking the inverse of the pricing function ($g = f^{-1}$) and inserting the current price of the option contract ($\bar{V}$) as input of the model, we can compute the volatility implied by the market:

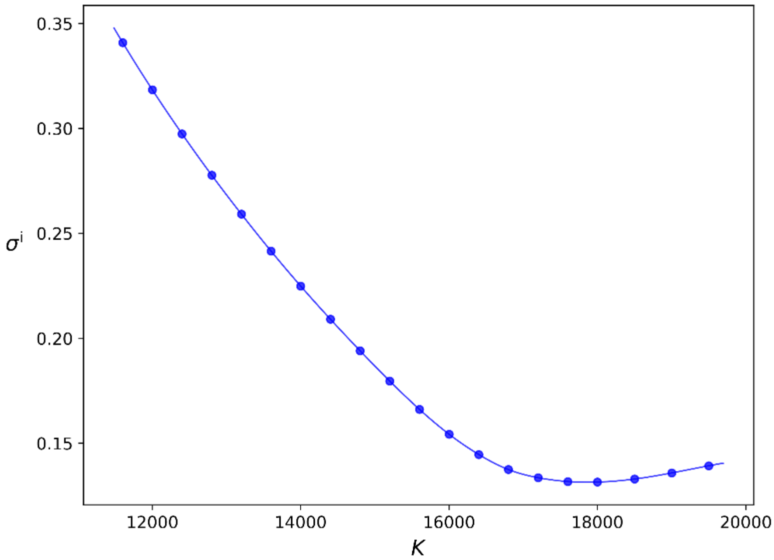
$$\sigma_{\bar{V}} = g(\bar{V}, \quad \cdot).$$

The groundbreaking work of Black and Scholes, together with Merton's contribution, has undeniably transformed the international financial system and supported the astonishing growth of options trading to unprecedented levels. The model is straightforward and provides an informative initial approximation, especially within the scope of setting up hedges for risk minimization. Attention must yet be drawn to applying this standard model for real-life investment decisions without making crucial considerations.

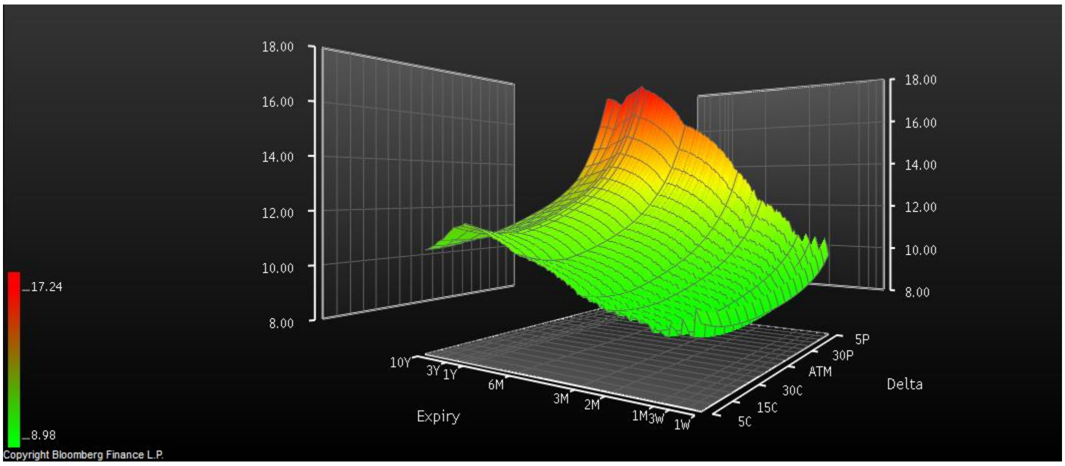
1. Firstly, in line with the empirical examination of option prices carried out by Macbeth and Merville (1979, p.1185): "*[...] the extent to which the BS model underprices (overprices) an in the money (out of the money) option increases with the extent to which the option is in the money (out of the money), and decreases as the time to expiration decreases*".

2. Moreover, the model assumes that the log return of the stock price follows a Wiener process, whose increments are independent, stationary, and normally distributed. In the real world, the assumption of normality does not encapsulate asymmetries and radical fluctuations in the stock market, such as crashes, leading to tail risk.

As we have examined in this section, options traders can observe the price of derivative contracts directly from the market and invert the Black-Scholes formula to calculate the implied volatility, which is widely used as a quoting convention for option prices. After the Crash of 1987, there was a reevaluation of fat-tail probabilities, which led to higher prices for deep OTM options and higher implied volatilities associated with them. When traders graphed the implied volatilities with respect to different strike prices, they detected a skewed pattern named "volatility smile".



The above graph displays the volatility smile based on the implied volatility for different strikes of European call options on the DAX in index points and identical expiry date. By graphing the implied volatility over a defined collection of maturities and exercise prices, traders can plot the implied volatility surface, which displays the volatility smile and the term structure of volatility together.



Setting aside its limitations, the Black-Scholes model delivers approximate option prices and stands as a sound foundation for more polished extensions due to its ease of understanding and the possibility of sorting out some of its inadequacies.

3.3 The Relation between Historical and Implied Volatility

After having expanded on the concept of volatility and differentiated between *HV* and *IV*, we begin this section with a presentation of results from academic literature on the predictive power of these two types of volatilities to forecast future volatility. Afterward, we are going to compare the S&P 500 (or SPX) and the VIX via descriptive statistical analysis.

Siu D. & Okunev J. (2009) investigate whether volatility persistence can be described in a 15-year dataset of intra-day data comparing implied volatility against a series of dependence models with short-range (ARMA, GARCH) and long-range (ARFIMA, RiskMetrics, and Random Walk) memory specifications for four exchange rates over horizons stretching from 1 day to 3 months. The empirical results suggest that significant incremental information is found in historical forecasts beyond the implied volatility information. Furthermore, within the historical forecasts, there is a considerable improvement from the use of high-frequency data rather than long-memory specifications, with the ARFIMA model producing the most accurate and consistent forecast for all horizons.

Bentes S. (2017) uses monthly data from BRIC countries to measure the informational content of implied volatility (*IV*) in explaining future realized volatility (*RV*) (H_1), in addition to testing for unbiasedness (H_2) and efficiency (H_3) of the estimator. They employ a static OLS regression and compare the results with an ADL (Autoregressive Distributed Lag) model to evaluate the dynamic relations between those variables. They find that:

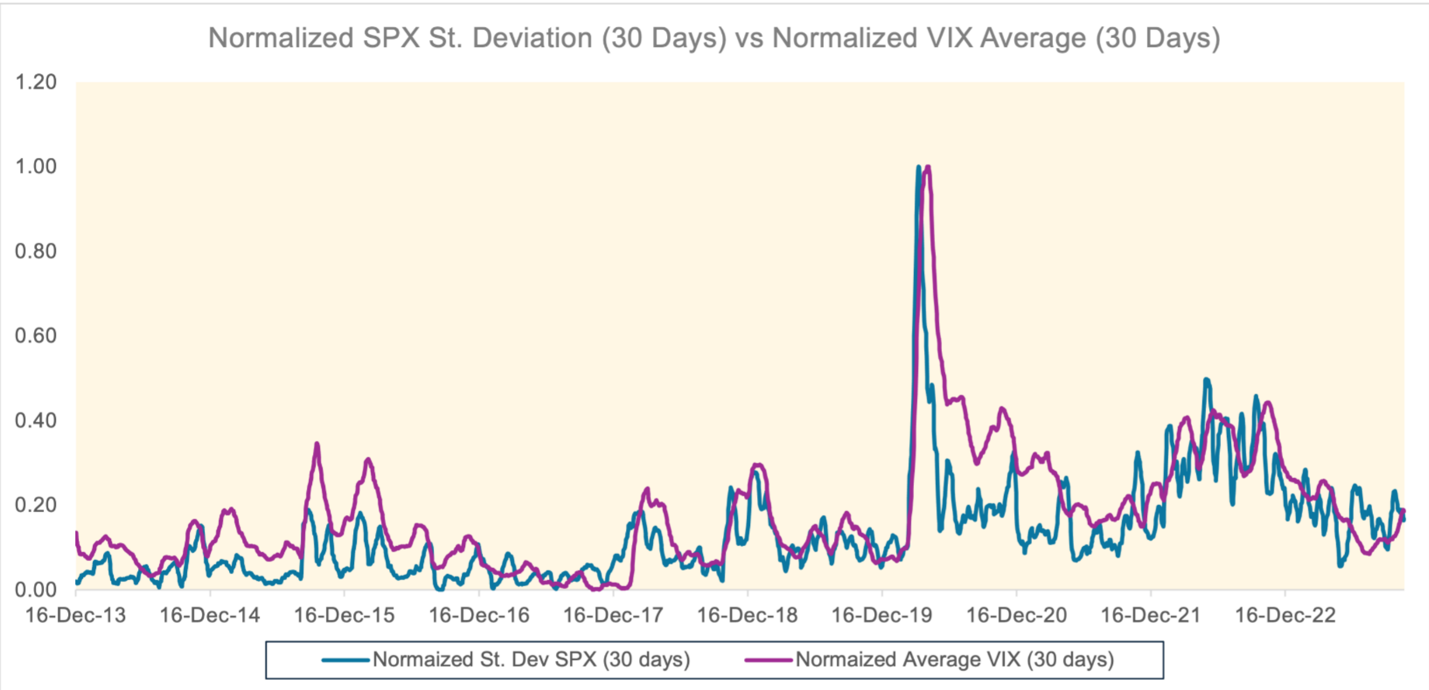
1. The regular method centered on the OLS regression suggests that *IV* has sufficient explanatory power for future *RV* in Russia and India, while in the ADL model, this holds for Brazil and China.
2. Both methods reveal that *IV* is an unbiased estimator of future *RV* in India.
3. Both methods indicate that *IV* is not an efficient estimator of future *RV*.

When we examined the risk measure Vega, we indicated that options could be a means of gaining exposure uniquely to the volatility of an underlying asset, for instance, through a strategy of the long (short) straddle. It can be crucial for traders to judge the realized volatility against the current level implied by the options market.

As anticipated, our analysis is a succinct comparison between the S&P 500 (SPX) and the CBOE Global Markets Volatility Index *(VIX)*, which is designed to be an up-to-the-minute estimate of volatility for the S&P 500 over the next 30 days and serves as a proxy of implied volatility. Indeed, the VIX is not computed using stock prices but with the midpoint of the S&P 500 Index option bid/ask quotes.

We employed two samples of 2,519 trading days for the S&P 500 and the VIX from 04.11.2013 to 03.11.2023. We computed the 30-day rolling standard deviation of the SPX and the 30-day rolling average of the VIX for better comparability, thus yielding 2,489 observations. Afterward, we normalized our datasets with a min-max feature scaling to bring all values into the range $[0, 1]$ for graphical purposes.

Firstly, we can visualize how the VIX has historically provided higher readings for volatility relative to the standard deviation of the S&P 500 Index. The trend of standard deviation rising to the same level or somewhat higher than the volatility reading provided by the VIX has been detailed exclusively during periods of extreme market turbulence, such as the Great Financial Crisis.

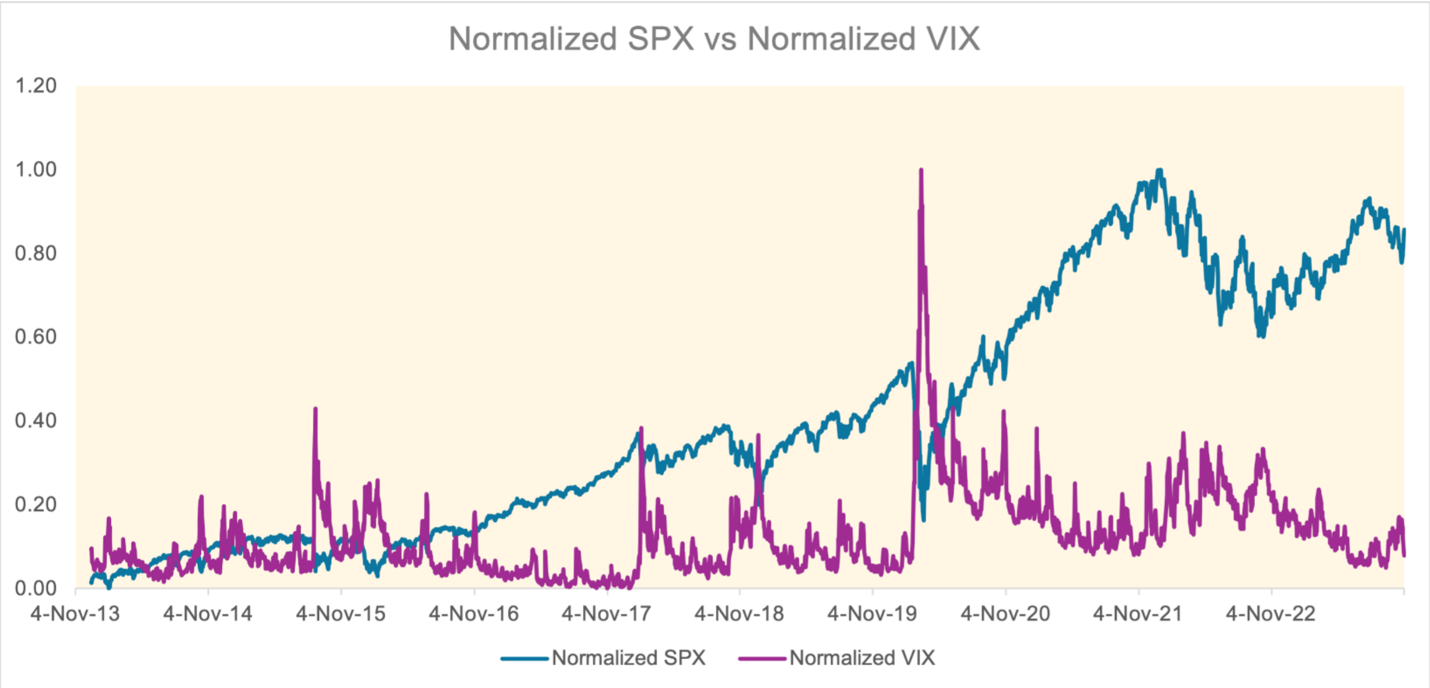


This point can be further corroborated by comparing the mean standard deviation of the SPX to the mean VIX over a 30-day window throughout the period analyzed.

St. Dev SPX (30 days)		*Average VIX (30 days)*	
Mean	**0.133**	**Mean**	**0.181**
Standard Error	0.002	Standard Error	0.003
Median	0.103	Median	0.142
Mode	N/A	Mode	0.181
Standard Deviation	0.120	Standard Deviation	0.144
Sample Variance	0.014	Sample Variance	0.021
Kurtosis	10.444	Kurtosis	7.138
Skewness	2.458	Skewness	2.055
Range	1	Range	1
Minimum	0	Minimum	0
Maximum	1	Maximum	1
Sum	330.996	Sum	449.880
Count	2489	Count	2489
	1		1

Secondly, option premiums tend to move inversely to the stock market. In bull markets, stock prices present lower volatility and lower implied volatility, as illustrated in the VIX, thus lower option premiums. When markets are moving sharply, volatility is generally higher, and premiums soar. Despite this financial intuition, the VIX has historically shifted in the same direction as the S&P 500. In the last ten years, the standard deviation of the SPX and the mean of the VIX present a correlation of $\rho = 0.2876$, confirming the previous statement. Consequently, traders insinuate that the VIX is defective as the two markets move in jointly.

	SPX	*VIX*
SPX	1	
VIX	0.287551	1



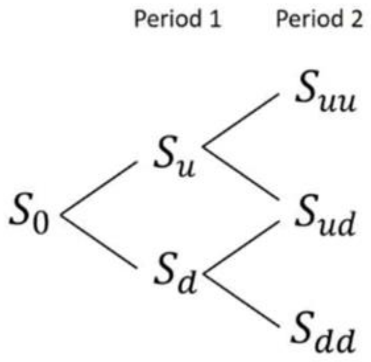
Overall, the VIX reflects efficiently the emotional state of the market. Traders can start to comprehend the VIX by comparing its price relative to price changes in the S&P 500. We can observe that, between 04/03/2020 and 16/03/2020, the VIX soared from 31.99 to 82.69 (an increase of 158.49%), reflecting the 23.77% drop in the SPX during the same timeframe.

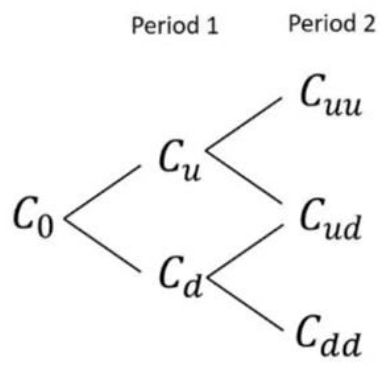
4 Option Pricing Models

Option pricing models are mathematical models that are used to calculate an option's theoretical value, which is an estimate of an option's fair price based on all available inputs. Variables such as the underlying asset's price, current market price, strike price, time to expiration, volatility, and interest rates are utilized as inputs in these models. Finance experts and investors can change their trading strategies and portfolios if they know the estimated fair value of an option. Popular models to value options are the Binomial model, the Black-Scholes model, its extension, the Heston model, and the Monte-Carlo simulation.

4.1 The Binominal Model

The Binomial Model is the basic option pricing approach, and it is used for American and European options. It estimates option prices using a discrete-time, step-by-step iterative approach. The concept is built on the idea that the underlying asset's price can either rise or fall during each time step, resulting in a binary tree structure of possible future underlying prices.





The Binomial Model is based on the following assumptions. The time to expiration is separated into discrete, evenly spaced intervals. Over a given period, the underlying asset price can only move up or down by a fixed percentage. The probability of the underlying asset price moving up or down is known and fixed. Transaction costs and taxes are ignored, and the interest rate is known and fixed.

Valuation of Put and Call Options in the Binominal Model

Valuing options through the Binomial Model requires the following steps:

1. *Construct a binomial tree*: This involves specifying the number of time steps in the model and the probability of the asset price moving up or down at each time step.
2. *Calculate Option Payoffs*: The option's intrinsic value is computed at every node until the final time step, representing the expiration date.
$$Call\ option\ payoff: \max(S(T) - K, 0)$$
$$Put\ option\ payoff: \max(K - S(T), 0)$$
3. *Work Backward*: Move backward through the tree, node by node, and calculate the expected value at each node. This involves discounting the expected future payoffs to the present value using the risk-free interest rate. This is called *backward induction*.
4. *Determine option value*: The option's value at the initial node is the model's estimated fair price for the option.

The Binomial Model can be used to value both American and European options. However, there is a substantial difference in the valuation of American options because of the buyer's early exercise option. The binomial model considers the possibility of early exercise by calculating the value of the option in each node in the tree, both the early exercise payoff and the expected payoff if the option is held until expiration. If the early exercise payoff of the nodes at one point in time exceeds the corresponding expected payoff, then the American option is exercised.

4.2 The Black-Scholes Model

A well-known option pricing model for derivative pricing is the Black-Scholes model. When it was launched in the early 1970s, the paper transformed the world of pricing financial derivatives and was the common tool for trading options. It operates in continuous time, as opposed to discrete-time models such as the binomial model and allows for an approximate analytical pricing of European options.

The Black-Scholes model relies on a set of assumptions. The model considers only European options. The price of the underlying asset follows a geometric Brownian motion (corresponding to a lognormal distribution for the price at a given point in time). The risk-free rate is known and remains constant over time until the expiration date. The volatility of the underlying asset price remains constant over time until the expiration date. There are no dividend payments on the underlying asset. There are no transaction costs for trading the underlying asset. There are no arbitrage opportunities in the market. Any fraction of the price of the underlying can be borrowed at the short-term interest rate. There are no limitations to short selling. It is important to notice that these assumptions are idealizations and often do not hold in real-world financial markets.

The Black-Scholes Formula

The value for a call option at time t is given by:

$$C_t = S_t N(d_1) - K e^{-r(T-t)} N(d_2)$$

The value for a put option at time t is given by:

$$P_t = K e^{-r(T-t)} N(-d_2) - S_t N(-d_1)$$

Where:

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

With the notations:

- S_t: The price of the underlying asset at time t
- t: Current date
- T: Expiry date of the option
- K: Strike price of the option
- r: Risk-free interest rate: interest rate without a default risk premium
- σ^2: Volatility of the underlying asset
- $N(\cdot)$: Cumulative distribution function for a normal (Gaussian) distribution. It is the probability that a random variable is less or equal to its input (i.e., d_1 and d_2) in a normal distribution. Thus, $0 \leq N(.) \leq 1$.

4.3 The Heston Model: Extension of the Black-Scholes Model

The Black-Scholes model delivers an approximate price for an option, but it has one significant shortcoming. The model assumes that the implied volatility remains constant. However, this assumption is inconsistent with real-world facts, in which implied volatility is affected by other factors. The volatility smile refers to the observed phenomena in which implied volatility changes with strike prices, generating a smile-shaped curve when plotted against strike prices and expiration dates. The volatility smile demonstrates that the market assigns varying degrees of implied volatility to options with different strike prices but the same expiration date. At-the-money options typically have lower implied volatilities than out-of-the-money or in-the-money options. This skew or grin shape in implied volatilities indicates traders' expectations of the possibility of extreme price swings, skewing option prices relative to what the Black-Scholes Model predicts.

Steven Heston created the Heston model in 1993 to provide a more realistic representation of this variable. Implied volatility is treated as a separate stochastic process in this model, specifically as an additional Geometric Brownian motion. This means that volatility can change over time, mimicking the observed fluctuations in financial markets more precisely.

The Heston Model differs from other stochastic volatility models in the following ways:

- It takes into account a possible correlation between a stock's price and volatility.
- It represents volatility as a reversion to the mean.
- It provides a closed-form solution, which means that the answer is derived from a known set of mathematical operations.
- It does not necessitate stock prices to follow a lognormal probability distribution.

The calculation is as follows:

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_{1t}$$
$$dV_t = k(\theta - V_t) + \sigma\sqrt{V_t} dW_{2t}$$

With the notations:

- S_t: price of the underlying asset at time t
- r: risk-free interest rate
- $\sqrt{V_t}$: stochastic volatility of the asset price
- σ: volatility of the variance
- θ: mean reversion level of the variance
- k: rate of reversion to θ
- dt: infinitesimal small positive time increment
- W_{1t}: Brownian motion of the asset price
- W_{2t}: Brownian motion of the asset's price variance

4.4 The Monte-Carlo Method

The Monte Carlo method, also known as Monte Carlo simulation (MSC), is a broad class of computational algorithms that obtain numerical results through repeated random sampling. The Monte Carlo simulation is widely used in finance and other fields and has recently become an important technique in finance for option pricing. MSC avoids complicated mathematics and has a simple conceptual and practical implementation. This method computes the value of an option with multiple sources of uncertainty and randomness, such as changing interest rates, stock prices, or exchange rates, and so on. The downside of these kinds of simulations is the excessive need for computational power and extensive computation times. Although the MSC is often used by financial institutions, other financial models, such as the Heston model or more elaborate models, can deliver more accurate option prices in certain cases.

The first step is to simulate the underlying asset's price paths. This can be accomplished through the use of various methods, such as the random walk method or the Cox-Ingersoll-Ross (CIR) model. The method chosen will be determined by the specific option being valued as well as the assumptions that the modeler is willing to make.

Secondly, after simulating the underlying asset's price paths, the next step is to compute the option's payoff at the end of each price path. This entails applying the option contract's rules to each price path and discounting the payoffs back to the present.

Finally, the option payoffs are averaged across all price paths. This provides an estimate of the fair value of the option. The greater the number of price trajectories simulated, the more accurate the assessment of the option's fair value will be.

5 Market Opportunities and Outlook

The following section examines the ideal time for an investor to enter the options market. The decision to enter this market is critical for investors, and the timing can have a significant impact on their outcome. This discussion looks at the factors that determine when it is most favorable for an investor to enter the options market, considering the dynamic nature of the financial landscape and the opportunities that arise under specific conditions.

Implied volatility is low and expected to increase

The implied volatility is a crucial factor to consider since an option's premium depends on the expectations of the market participants. The implied volatility depends on the uncertainty based on the volatility of the underlying asset price as well as on the general market sentiment. An option's premium will be higher if the implied volatility in the underlying and the market is high, providing a higher probability for prices to fluctuate and increasing the option's payoff. On the other hand, if the implied volatility is low, the option's premium is cheaper and provides the investor with a favorable purchase opportunity.

Similar to economic factors, implied volatility evolves in cycles. Periods of high volatility are followed by periods of low volatility, and vice versa. Implied volatility is likely to revert to its mean when it reaches extreme highs or lows. Understanding whether implied volatility is low or high helps in the selection of an appropriate investment plan.

Regarding implied volatility, an investor should think about buying inexpensive options and selling overpriced options. Selling should be considered when observing options trading at high implied volatility levels and far in-the-money or anticipating a fall in implied volatility with minor upside potential in the option. Option premiums become less desirable for purchase and more appealing to write as they get more expensive. Buying should be explored when options are trading at low implied volatility levels or when implied volatility is predicted to climb.

When to enter the path-dependent options market

Entering the path-dependent options market can also be advantageous for investors in some instances, notably when anticipating major price fluctuations in the underlying asset.

The events triggering the fluctuations could be company-specific, such as earnings releases, product launches, or mergers and acquisitions, or market-wide, such as economic indicators, geopolitical developments, or important policy decisions. Using path-dependent options strategically allows investors to navigate periods of high uncertainty and capitalize on emerging trends. As previously noted, higher uncertainty frequently leads to increased implied volatility, which drives up option premiums.

The increased volatility creates a favorable environment for investors as it potentially enhances the profitability of their options positions, offering greater potential return on investment during uncertain times.

Advantages and Disadvantages of path-dependent options

The possibility for larger profits is one of the primary benefits of path-dependent options. These options can provide better returns than ordinary options if an investor correctly predicts the tendency of the underlying asset's price movement. Furthermore, path-dependent options can be useful tools for hedging against a variety of risks. For example, investors can tailor these options to handle specific risks in their portfolios, resulting in a flexible hedging strategy. Some path-dependent options also provide protection against the risk of selling at an unfavorable time. This feature gives investors an additional possibility to earn a bigger return without having to sell at optimal prices and risk missing out on superior profit opportunities.

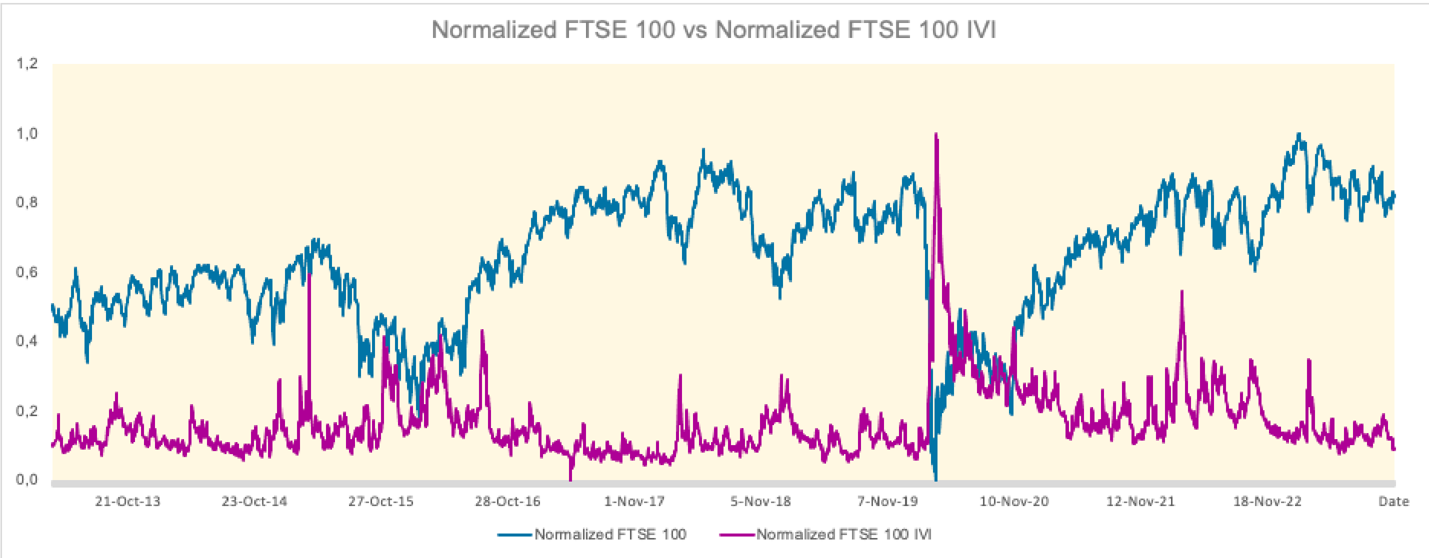
Regarding the disadvantages, path-dependent options are more difficult to price than conventional options since the payoffs are contingent on the future movement of the underlying. Pricing these possibilities requires sophisticated mathematical models, and the complexity grows with the complexity of the path-dependent features chosen. This intricacy might be an impediment for certain investors who lack the skills or information to determine the fair value of these options appropriately. Additionally, some path-dependent options increase the payoff and the probability of a payoff significantly. Therefore, path-dependent options are more expensive than ordinary options, and the payoff must first cover the premium expense before a profit can be realized.

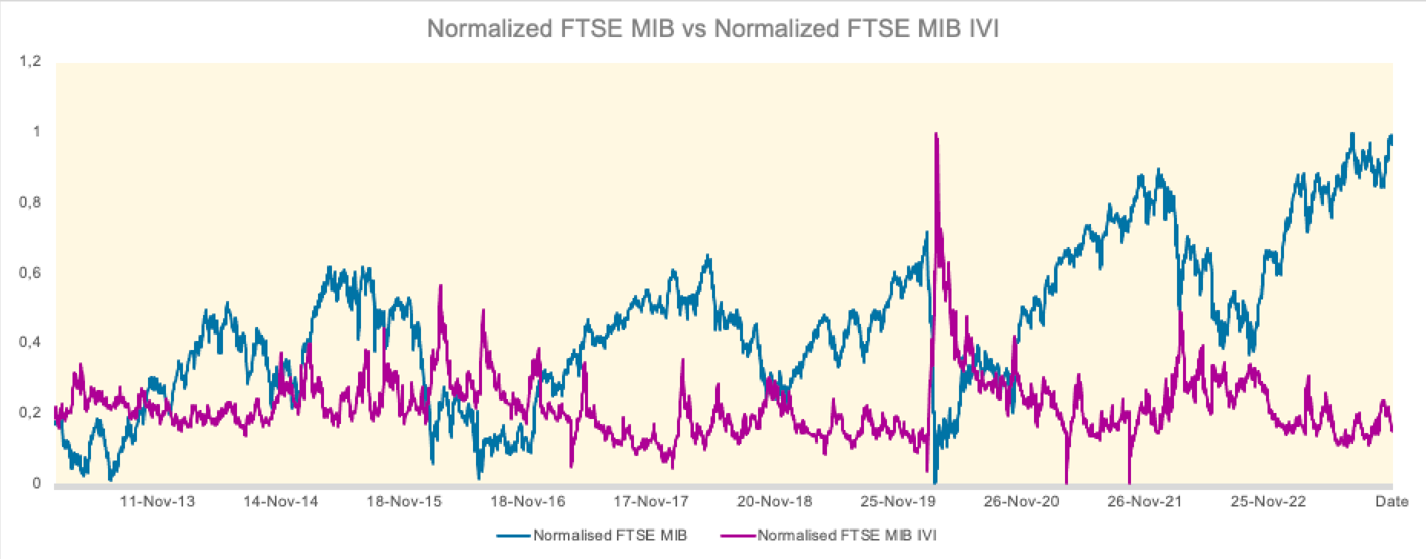
5.1 The current implied volatility level

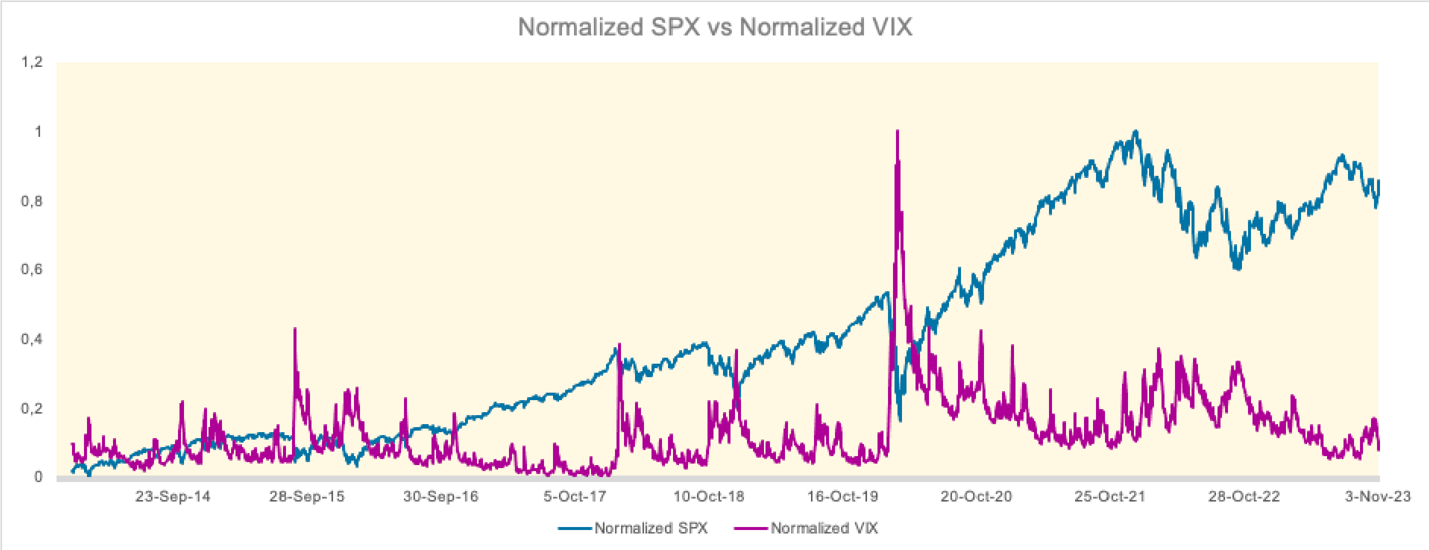
We analyzed the prevailing level of implied volatility by examining the three implied volatility indices FTSE 100 IVI, FTSE MIB IVI, and VIX.

The FTSE 100 IVI is an index that assesses the interpolated annualized implied volatility over 30 days based on the underlying FTSE 100 index. Similarly, the FTSE MIB IVI measures implied volatility related to the underlying FTSE MIB index, the stock market index for the Italian Exchange. Expected volatility is derived from the market prices of out-of-the-money options, with each option's price reflecting the market's anticipation of future volatility. Likewise, the VIX, also known as the CBOE Volatility Index, is a measure of market expectations and implied volatility for the S&P 500 index over the next 30 days. It represents investors' sentiment and their perceptions of future market volatility.

The three graphs presented below depict the normalized indices and their corresponding implied volatility spanning from January 2013 to June 2023. Generally, the stock indices tend to increase during periods of low implied volatility, which can be inferred from the low uncertainty and positive sentiment in the market. This inverse relationship between the index and implied volatility arises from the fact that in periods of low volatility and market calmness, people perceive less risk and are more inclined to invest in stocks, thereby driving up indices like the S&P 500. Conversely, in highly volatile contexts, individuals become more risk-averse, feeling endangered by market fluctuations, and are less willing to invest, leading to a decline in the value of indices. Therefore, the inverse relationship between the stock market index and the implied volatility index reflects investors' changing perceptions of risk and their behavior in response to market conditions.







The COVID-19 pandemic had a significant impact, driving the stock market indices sharply down and increasing the implied volatility vastly. Since the initial shock, stock markets recovered, and the implied volatility shows a clearly decreasing trend. In 2023, the market indices did not find a clear trend, but implied volatility decreased and is currently at a comparably low level. This decline in implied volatility was not favorable for investors holding options in their portfolio and subsequently implies a loss in the option's value regarding the implied volatility component. Moreover, the indecisive market did not favor options that depend on a clear trend for higher payoffs. Conversely, the current situation opens a window for investors seeking an opportunity to enter the option market at a lower premium than in the past three years.

5.2 Opportunities for path-dependent Options in 2024

The current market sentiment measured by the implied volatility indicates a low uncertainty, but the coming year is likely to offer several potential opportunities for investors to profit from path-dependent options. Financial markets are expected to offer distinct opportunities with high volatility and amplified uncertainty due to presidential elections, geopolitical situations, and a potential recession.

One event of particular significance will be the US presidential elections, scheduled for November 5th, 2024. The candidates have not yet been announced, but the main contenders are expected to be incumbent President Joe Biden for the Democratic Party and former President Donald Trump for the Republican Party. These elections will introduce considerable volatility as both Biden and Trump are polarizing figures with very divergent political agendas. In addition, after Trump's victory in 2016 against Democratic nominee Hillary Clinton, financial markets fell sharply on election night due to the surprising outcome and the uncertainty related to the president's policies, which announced the US exit from the Paris Climate Agreement and questioned the North American Free Trade Agreement (NAFTA). However, the stock markets recovered on the following day and rallied strongly during the remaining year. This led to an increase in economic uncertainty, exerting a strong impact on the markets. There is a risk that with this election, the situation will repeat itself and lead to a considerable increase in volatility in the markets, offering opportunities for path-dependent options to profit from the market changes.

Moreover, the gas crisis in Europe is probably not over yet. After the painful increase in energy prices in 2022, we have seen a steady decline this year, thanks to a combination of factors such as mild weather and a sharp drop in demand. Although EU reserves are at full capacity, this may not be enough. However, gas markets are becoming increasingly risky: gas and liquified natural gas prices are increasingly volatile and influenced by global factors. One factor of paramount importance is the conflict between Israel and Gaza, comprising the risk of dragging neighboring gas-producing countries into the conflict and causing a major conflict in an already fragile constellation. The overall uncertainty related to future events that might affect gas supply makes it extremely difficult to predict how gas supply will balance with demand and how prices might vary. Europe now has fewer alternatives to obtain additional gas in case of need. There could be a greater demand for fuel if the coming winter is much colder than expected or if Russia, which still supplies gas via pipelines to a limited number of European countries, stops exporting. According to the Timera Global Gas Report, prices could rise or fall from current levels, but the trend in the European gas market for both the current winter and 2024 shows an asymmetric risk of rising prices. An increase in gas prices would further worsen the already delicate competitive situation of the gas-intensive industry and delay the normalization of inflation. Moreover, the Russia-Ukraine war is tedious and might bring changes in 2024, as well as the irritable situation in the South China Sea around Taiwan.

Finally, the likelihood of a global recession cannot be ruled out, especially after the repeated interest rate hikes by many central banks over the past two years. In the US, many leading indicators that can predict GDP trends have weakened, including ISM manufacturing. In addition, the government bond yield curve remains inverted, indicating the likelihood of a short-term recession in the US. Historically, whenever there is an inversion of the curve, a recession occurs within 18 months, which is why many analysts, since the inversion of the curve in 2022, have assumed a recession in 2023 that has not yet occurred. Therefore, it is reasonable to assume that this could happen in 2024. Outside the US, the Eurozone economy has already contracted for two consecutive quarters. A long and deep global recession is not expected, but rather a mild one in 2024. Despite the slowdown in inflation this year, economies have not yet paid the price in terms of job losses or broader economic contraction. Indeed, in the first quarter, GDP growth accelerated in the US, Europe, and China. In all developed market economies, the unemployment rate remains well below its pre-pandemic level. The price of oil has fallen by almost 30% compared to a year ago, showing a moderate deflationary effect. Some investors are betting on rate cuts in the second half of the year, but this prediction may be too early. The authorities are unlikely to give up the fight against inflation unless inflation returns to its target level or the labor market situation starts to show more worrying signs. Citigroup analysts predict that the US economy will likely enter a recession in 2024, citing historical precedents of periods of high inflation and tight labor markets. Analysts argue that the elimination of high inflation and tight labor markets has historically led to recessions and that similar patterns could reoccur in the next cycle. A significant moderation in wage and price growth is expected, while presenting a convincing narrative for a soft landing scenario remains difficult.

Analyzing these scenarios, it becomes clear that there is potential for increased market volatility in 2024. This situation appears to be introducing interesting opportunities for profit by utilizing path-dependent options. In particular, if there will be a recession in 2024, accompanied by a possible increase in market volatility, it may be advantageous to consider purchasing a floating lookback put option.

During periods of high volatility, price fluctuations can become more extreme and unpredictable. This specific option, based on the minimum price reached by the asset during the option period, offers a significant advantage as prices can vary widely. If the market is expected to collapse, the put lookback presents the possibility of profiting from substantial price movements, generating higher gains than traditional options. Moreover, during periods of volatility, markets can react unpredictably to sudden news and events. The lookback put offers flexibility to take advantage of sudden price changes, adapting to unexpected changes in market performance without missing the optimal moment for execution.

In line with the strategy proposed above, an alternative path-dependent option is the Asian option, which profits from a situation with high volatility and possible downturns. The final payoff depends on the average price of the underlying during the specified period. Since volatility is an indicator of unpredictable scenarios, it could be chosen by investors to make a profit despite possible large market fluctuations, bought at a lower premium than common options. In particular, an average put option takes advantage of a possible future market slump, profiting even if the market contracts in an unpredictable manner and with sudden trend changes.

In conclusion, there are several opportunities to invest in path-dependent options based on different market forecasts for the future year. Despite the decreased uncertainty in 2023, we expect multiple opportunities for investors to profit from volatility and market trends in 2024. This scenario gives a chance to effectively leverage these financial instruments, profiting from the market environment and the beneficial characteristics of path-dependent options.

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